

Building a Sovereign Benchmark

The Limits of Private Coordination*

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Abstract

Public debt supplies liquidity and collateral services only if institutions coordinate on using the same claim. We study this coordination when use carries a recurring cost and a benefit that rises with other users. Above a quality threshold, two positive-adoption equilibria exist, and under best responses the initial user base decides between collapse and high adoption. Under logit choice, as the population grows and noise vanishes, the long-run distribution selects high adoption only above a strictly higher threshold. Debt issuance improves market depth but eventually dilutes backing per claim. This trade-off creates a range of debt levels supporting the liquidity premium of public debt in macroeconomic models and a narrower range supporting its long-run selection. At the boundary of the wider range, the premium is discontinuous. A matching transfer that corrects the adoption externality makes stationary selection maximize network surplus. Transfers can also help launch a network without permanent support.

Keywords: Public debt; liquidity services; safe assets; network externalities; equilibrium selection.

JEL codes: C72, C73, E42, E44, H63.

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1 Introduction

Government bonds provide services beyond their cash payoff. Banks and dealers use them as collateral to obtain short-term funding through repurchase agreements, or repos. Institutions can also post them to meet initial margin requirements where they are eligible, take positions in them to hedge interest-rate risk, or borrow them to fulfill securities delivery obligations. These activities depend on counterparties that are willing and able to transact in the same claim. A dealer's ability to finance a position, for example, is more valuable when a broader set of lenders accepts the security as collateral. Common use therefore creates opportunities that the claim's promised repayment alone does not provide. The service is produced by the users, not owned by the asset.

These services are central to the macroeconomic role of public debt. By supplying liquidity, public debt can relax private financing constraints ([Woodford, 1990](#); [Holmström and Tirole, 1998](#); [Aiyagari and McGrattan, 1998](#)). The resulting liquidity premium shapes the optimal quantity of government debt ([Angeletos et al., 2023](#); [Bassetto and Cui, 2024](#)). That premium is one reason the government's borrowing rate can sit below the growth rate, which changes the fiscal cost of carrying debt ([Blanchard, 2019](#); [Reis, 2022](#); [Kocherlakota, 2023](#)). Liquidity and safety services also matter for the valuation of the outstanding debt stock ([Krishnamurthy and Vissing-Jorgensen, 2012](#); [Jiang et al., 2024c](#); [Brunnermeier et al., 2024](#)). Whether these services persist as debt expands therefore matters for both private finance and public borrowing. This question is particularly relevant for U.S. Treasuries given the projected growth in public debt.¹ In these models, the liquidity of public debt is taken as given: the claim is fully liquid, or a demand for it is specified directly.

In practice, sustaining common use requires institutions to maintain valuation systems, collateral-management arrangements, and settlement procedures. In this paper, adopting a government security means maintaining the capacity to use it in these activities, rather than simply buying and holding it. Each institution weighs the ongoing cost of that capacity against the opportunities supplied by other users. An established reference consequently benefits from its installed base, while reliable repayment alone may not justify participation in a sparsely used claim. A related coordination force arises for reference rates: [Duffie and Stein \(2015\)](#) explain how the liquidity of LIBOR-linked markets encouraged participants to continue trading instruments tied to LIBOR. We examine this participation margin for government securities, treating their common use as an equilibrium outcome. Our focus is on when the network supplying these services can sustain itself, and which adoption outcome prevails as institutions repeatedly

¹In its February 2026 baseline, the Congressional Budget Office projects federal debt held by the public to rise from 101 percent of GDP in 2026 to 120 percent in 2036 ([Congressional Budget Office, 2026](#)).

reconsider their participation.

To study this coordination problem, we consider institutions choosing between using a government claim as their reference and retaining an outside convention. Institutions differ in their recurring implementation costs. The benefit of adoption rises with the share of other users and with benchmark quality, which combines market depth with the fraction of the real payoff retained by a holder. We first hold quality fixed to study adoption, and then examine how debt supply affects quality. An equilibrium requires the share of institutions willing to adopt to equal the share actually using the reference. The model gives a microfoundation for when the service exists, not for how much each unit of it is worth.

The first result characterizes these equilibria. If nobody uses the reference, no institution wants to pay its implementation cost alone, so zero adoption is always an equilibrium. When quality is too low, it is the only equilibrium. Once quality reaches a threshold, a positive network can sustain itself. Above that threshold, there are exactly two positive equilibria, one with fewer users and one with more users, in addition to zero adoption. The two positive equilibria meet at the threshold and disappear below it. Thus sufficiently good quality makes a functioning network possible, but does not make its adoption inevitable.

We next specify how institutions revise their decisions. Each institution receives opportunities to reconsider adoption independently of the others. Between opportunities, its action remains unchanged. When an opportunity arrives, it compares the benefits of using the current network with its implementation cost. We use a logit choice rule: the more profitable action is more likely, but either action can be chosen when revision noise is positive. As this noise vanishes, choices approach best response.

Without revision noise, the adoption network follows a deterministic adjustment path. The zero-adoption and high-adoption equilibria are locally stable, whereas the low-adoption equilibrium is unstable. An initial user base below the low-adoption equilibrium shrinks; one above it moves toward the high-adoption equilibrium. The low-adoption equilibrium is therefore the critical mass separating eventual abandonment from convergence to the high-adoption equilibrium. An established reference can consequently persist under conditions in which a smaller initial user base would fail to grow.

With a finite population and positive revision noise, the long-run frequencies of adoption are described by a unique stationary distribution. This distribution is a Gibbs measure: an adoption configuration receives a weight that increases exponentially with its potential. The potential is a summary of private incentives. When one institution changes its action, the change in potential equals that institution's payoff gain. As the population grows and revision noise vanishes, the stationary distribution concentrates on the equilibria with the highest potential. Only zero adoption and the high-adoption equilibrium are candidates for the potential maximizer.

We then characterize when the high-adoption equilibrium wins this potential comparison. The high-adoption equilibrium is the unique potential maximizer if and only if quality exceeds a threshold. This threshold lies strictly above the quality needed for a positive equilibrium to exist, and strictly below twice that level. The reason is that sustaining a network at a given size and overcoming the private incentives against adoption at smaller sizes are different requirements. At the existence threshold, the positive equilibrium still has lower potential than nonadoption. Further improvements in quality eventually reverse that ranking. Between the two thresholds, an established benchmark can persist, but it is not where a noisy economy spends most of its time.

To translate these results into debt supply, we suppose that benchmark quality first rises and then falls with the amount of eligible debt. A small issue offers too little market depth. A larger issue improves depth, but eventually spreads the resources backing the claim across more promised payments. The debt levels supporting a positive equilibrium therefore form a feasibility corridor. The stronger requirement for stationary selection produces a narrower coordination corridor. A debt level may support an established network without placing that network in the long-run selected regime. [Brunnermeier et al. \(2024\)](#) and [He et al. \(2019\)](#) also find that debt size works in both directions, the first through a service flow spread over a larger stock and the second through market depth and rollover risk.

We next discuss welfare implications. The quality needed for positive net surplus is below the quality needed for a positive network to exist. Between those two, a network would create surplus, yet none can exist without support. A policy exercise holds debt and backing fixed and examines a transfer that rewards adoption in proportion to the existing network. An institution considers the benefit it receives from joining, but not the benefit its participation creates for other users. Under nondistortionary transfer finance, a payment equal to this external benefit aligns the potential with aggregate network surplus. The same stationary selection mechanism then implements a surplus-maximizing adoption share.

The transfer can also help launch a network without being needed permanently. When benchmark quality is above the existence threshold, a high-adoption equilibrium can sustain itself, yet a small initial user base may fail to reach it. Under noiseless renewal, a matching transfer that makes this initial network grow eventually raises adoption above the critical mass required without support. The government can then withdraw the transfer, and adoption converges to the unsubsidized high-adoption equilibrium. This distinguishes launching a network from maintaining it: temporary support helps build the user base, while the network's own benefits sustain participation once that base is sufficiently large.² At fixed benchmark

²[Crouzet et al. \(2023\)](#) find that a temporary cash shortage in India left electronic-payment adoption permanently higher.

quality, the intervention changes which equilibrium is reached without requiring a permanent subsidy to maintain it.

Finally, a pricing extension separates the claim's cash payoff from the service supplied by its adoption network, the cash-flow plus service-flow split of [Brunnermeier et al. \(2024\)](#). We follow an operating network along the high-adoption branch to the upper debt level at which positive equilibria cease to exist. Beyond that boundary only nonadoption remains, and the price falls by the discounted value of the lost service. [Chen et al. \(2025\)](#) document historical cases in which the dominant supplier of safe debt later lost that position. In the model that position can be lost without a jump in backing.

Related literature. A macroeconomics literature treats government debt as an asset that eases private liquidity constraints. [Woodford \(1990\)](#), [Holmström and Tirole \(1998\)](#), and [Aiyagari and McGrattan \(1998\)](#) give public debt a role as private liquidity, or as a store of value when private markets are incomplete. [Angeletos et al. \(2023\)](#) and [Bassetto and Cui \(2024\)](#) study how much of that debt a government should supply when it carries a liquidity premium. [Bassetto and Cui \(2018\)](#) study fiscal backing when interest rates are low. [Blanchard \(2019\)](#), [Reis \(2022\)](#), and [Kocherlakota \(2023\)](#) examine the fiscal arithmetic of rolling debt over when the interest rate is low relative to growth. [Dong et al. \(2026\)](#) show that persistent primary deficits can produce multiple steady states and select a low-interest, low-productivity equilibrium, so an interest rate below growth need not be a free lunch. [Kiyotaki and Moore \(2019\)](#) let assets differ in how much of them can be resold in a period. [Jiang et al. \(2024a\)](#) determine government debt capacity when debt has a convenience yield and the issuer can default. [Jiang et al. \(2026\)](#) show that financing accumulated deficits with short-term debt lets later governments carry out an optimal tax plan even when inherited debt is high.

Other work studies the same debt as a safe asset. [Farhi and Maggiori \(2018\)](#) show that issuing more reserve assets exposes the issuer to a confidence crisis. [Caballero and Farhi \(2018\)](#) study a shortage of safe assets that produces a deflationary trap at the zero lower bound, where more public safe debt raises output. [Jiang et al. \(2024b\)](#) connect the safety of dollar debt to global financial conditions. [Brunnermeier et al. \(2024\)](#) price a safe asset as the sum of its cash flows and a service flow from retrading, and the value of that service can fall as more debt is issued. [Chen et al. \(2025\)](#) find that dominant suppliers of safe debt have lost that role when fiscal backing weakened. These papers take the liquidity of public debt as given, or they derive it from the asset's payoff. We derive it from institutions' decisions to use the claim.

A related line of work makes the acceptability of an asset the result of individual choices. [Kiyotaki and Wright \(1989\)](#) have an asset become a medium of exchange because others accept it. [Lester et al. \(2012\)](#) let agents pay to recognize an asset: recognition by more agents makes

it more liquid, which raises its price and the return to recognition, and multiple equilibria can result. In [Asriyan et al. \(2019\)](#), adverse selection and resale make liquidity today depend on expected liquidity tomorrow, and sunspots select among self-fulfilling equilibria. [Cui and Radde \(2020\)](#) obtain the liquidity of private claims from investor participation in a search market, with government bonds filling the gap when that participation is low. [Atkeson et al. \(2015\)](#) study banks' entry into and exit from an over-the-counter market; after large negative shocks, dealers can exit more than is socially optimal.

International macroeconomics asks the same question about currencies and safe assets. [Matsuyama et al. \(1993\)](#) have national currencies circulate only locally or as an international currency, and use an evolutionary approach to select among those equilibria. [Chahrour and Valchev \(2022\)](#) show that saving in an asset and using it to finance trade reinforce each other, which produces several stable steady states, each with a different dominant asset. [Coppola et al. \(2025\)](#) let liquidity provision and the supply of safe bonds drive currency use, with transitions between regimes that depend on safe-asset supply. For a single sovereign claim, the single-peaked cost distribution delivers a sharp threshold for a positive network to exist and a higher threshold for long-run selection.

The adoption model belongs to the literature on network externalities and interdependent discrete choices. [Katz and Shapiro \(1985\)](#) and [Farrell and Saloner \(1985\)](#) establish how compatibility and the choices of other users shape adoption incentives. [Rohlf's \(1974\)](#) has the same critical-mass logic: zero use is stable, and the smaller positive equilibrium is not. [Farrell and Saloner \(1986\)](#) show that the users already in place can lock in the existing standard. [Brock and Durlauf \(2001\)](#) study self-consistent aggregate choices and multiplicity with social interactions.³ We impose a single-peaked cost density to obtain a sharp existence threshold, and compare that threshold with the one generated by stationary logit selection.

Our adjustment mechanism uses tools from potential games and evolutionary dynamics. The exact-potential property follows the framework of [Monderer and Shapley \(1996\)](#); [Sandholm \(2001\)](#) and [Sandholm \(2009\)](#) develop potential games for large populations. [Blume \(1993\)](#) connects noisy individual revisions, invariant distributions, and equilibrium selection. [Sandholm \(2010\)](#) studies large-population and small-noise limits of stationary distributions and shows that selection can depend on the noisy response rule. The Gibbs formula and the general potential-selection principle are therefore methodological foundations, rather than new selection criteria. We apply them to a population with fixed heterogeneous costs, prove the approximation from finite action profiles to the continuum potential, and use one revision protocol to distinguish installed-base dependence on finite horizons from its stationary selection. The existence bound-

³An additive, fixed-heterogeneity formulation is [Gordon et al. \(2009\)](#), who analyze binary adoption with a distribution of individual willingness to pay and a benefit that increases with the adoption share.

ary does not depend on choosing logit as the revision rule; the coordination boundary is specific to the maintained selection mechanism. [Kandori et al. \(1993\)](#) and [Young \(1993\)](#) select by rare mutations, and the expected wait grows exponentially with population size, so a finite horizon need not reach the stationary outcome. [Ellison \(1993\)](#) shows that under uniform matching these waits are long enough for history to determine play, while interaction among a few neighbors shortens them. Adoption benefits here depend on aggregate use, which is the uniform case. [Carlsson and van Damme \(1993\)](#) and [Morris and Shin \(2003\)](#) select from incomplete information about payoffs. [Frankel and Pauzner \(2000\)](#) study sectoral choice with external increasing returns, where workers switch only at random times. A Brownian shock that can make either sector dominant selects a unique path. Macro-finance models more often resolve multiplicity with a sunspot ([Azariadis et al., 2016](#)) or with a global game ([He et al., 2019](#)). The revision protocol used here keeps those equilibria and adds a criterion for where the economy spends its time, which is what separates the two thresholds.

A close sovereign-debt comparison is [He et al. \(2019\)](#). They determine safe-asset status through investors' coordination across sovereign debt markets, with debt size affecting both market depth and exposure to rollover risk. The trade-off between depth and backing is therefore not specific to the present paper. Here there is one eligible reference claim, its payoff retention is a primitive, and institutions coordinate on using it rather than on refinancing its issuer. In [Cole and Kehoe \(2000\)](#), lenders coordinate on refinancing and a failed auction leads to default. [Aguiar et al. \(2022\)](#) add issuance at depressed prices and sudden stops. In [Lorenzoni and Werning \(2019\)](#), fear of default raises interest rates, debt then grows faster, and default eventually follows. Here adoption does not determine the claim's cash repayment, so the price drop at the edge of the feasibility corridor is a loss of service rather than a default. Our question is whether the use of that claim as a common reference can be sustained, and how the set of supporting debt levels changes when a stationary selection requirement is added to equilibrium existence. The single-peaked quality assumption makes the trade-off between depth and retention explicit without modeling a rollover game. Other financial-reference models study different benefits from common use. [Doepke and Schneider \(2017\)](#) explain a dominant unit of account through noncontingent contracts and the reduction of relative-price risk along credit chains. [Duffie et al. \(2017\)](#) study informational benchmarks that improve price transparency, search, and matching in over-the-counter markets. [Yuan \(2005\)](#) models a benchmark security as a way to trade systematic risk apart from security-specific risk. [Dittmar and Yuan \(2008\)](#) find that sovereign bonds play this role for corporate bonds in emerging markets. Our reference is a security around which institutions organize their activities, not a published price signal; neither information aggregation nor denomination choice drives the adoption equation. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) document the value of liquidity and safety services in Treasury

yields and their response to supply. Our pricing extension takes the valuation of such a service as given. Its content is the loss of that service when the positive-adoption branch ends, rather than a new pricing formula or an empirical estimate of convenience yields.

Finally, the corrective-transfer result is an application of the stochastic evolutionary implementation approach of [Sandholm \(2007\)](#). That paper shows how Pigouvian prices can produce efficient long-run behavior under logit revisions despite limited information and multiple equilibria. In our setting the correction takes the simple form of a payment proportional to aggregate adoption. It aligns the adoption game's potential with network surplus, without eliciting individual cost reports. [Dybvig and Spatt \(1983\)](#) treat adoption externalities as a public good and note a role for intervention when one equilibrium Pareto-dominates the others. [Benigno and Fornaro \(2018\)](#) show that investment subsidies can lift an economy out of a stagnation trap by shifting growth expectations. A temporary matching transfer works here through the installed base: it raises adoption above the critical mass, after which the network sustains itself. [Sákovics and Steiner \(2012\)](#) instead study how to target subsidies across heterogeneous agents to achieve successful coordination at minimum cost under strategic uncertainty. We do not solve that targeting problem: our exercise holds debt and backing fixed, uses a common matching rate, and characterizes its effect on the adoption thresholds. [Hu and Zhang \(2025\)](#) give a different reason why institutions may not use a public facility: from 2008 to 2010 banks avoided the discount window but borrowed through the Term Auction Facility, which had the same requirements, because the two carried different stigma. Here the obstacle is a small user network, which the matching transfer addresses directly.

We use a coordination model to ask a macroeconomic question: when does public debt supply the liquidity service that this literature assigns to it? The answer is two ranges of debt. Inside the wider range a positive network can exist, and at its edge the convenience value is discontinuous. Inside the narrower range that network is the long-run outcome. A temporary transfer can start a network that then sustains itself. The analysis does not replace models of optimal debt. It gives the conditions under which the liquidity service they assume is supplied.

Section 2 sets out the environment. Section 3 characterizes the equilibria and their potential. Section 4 introduces the logit rule, applies it first to continuum renewal and then to finite-population stationary selection, and characterizes the selection threshold. Section 5 studies debt corridors, corrective transfers, and convenience value. All proofs are in Appendix A; a separate finite-horizon consistency result connects the two population descriptions. Appendix B allows the cost density to be positive at zero.

2 Environment

Debt and benchmark quality. The real face value of the eligible government claim is $x > 0$, and $\varphi > 0$ measures the real resources backing those claims. Each unit promises a real payment normalized to one. Let $s(x; \varphi) \in (0, 1]$ denote its *payoff retention*: the fraction of this unit payment delivered to its holder.⁴ Let $m(x) > 0$ measure effective market depth; its scale incorporates infrastructure and legal usability. Define *benchmark quality*, also referred to as *benchmark strength*, by

$$b(x; \varphi) = m(x)s(x; \varphi). \quad (1)$$

Quality combines ease of use with the value retained by the claim. The two factors follow the distinction between an asset's liquidity and its safety in [Geromichalos et al. \(2023\)](#). The adoption analysis takes these functions as given; Section 5 specifies how they vary with eligible debt supply.

Institutions. There is a unit mass of institutions. Each chooses $a_i \in \{0, 1\}$, where adoption means operating with the government claim as its reference; the outside convention yields a payoff normalized to zero. Costs have cross-sectional distribution F . An institution's c_i is the recurring cost of maintaining the operational capacity to use the claim as its reference, relative to the outside convention. Examples include operating valuation and settlement systems and retaining staff or external services for collateral monitoring, margin management, and reconciliation. These activities continue after initial setup.⁵ We model the reference-specific maintenance burden as a cost of keeping this capability active, rather than a fee for each transaction.

Let $A = \int a_i di$ denote aggregate adoption. An adopter encounters compatible counterparties at rate $m(x)A$ and obtains a benefit scaled by payoff retention $s(x; \varphi)$. Its gross benefit is therefore $b(x; \varphi)A$. This is a thick-market externality of the kind in [Diamond \(1982\)](#): the return to participating rises with the number of other participants. Writing $b = b(x; \varphi)$, the payoff difference of adoption is

$$u_i(1; A) - u_i(0; A) = bA - c_i. \quad (2)$$

The adoption analysis initially holds $b > 0$ fixed.

Assumption 1 (Single-peaked cost distribution). *The distribution has support $[0, \infty)$, $F(0) = 0$,*

⁴Retention below one can come from partial default or from inflation. The parameter φ is the fiscal backing of the claim. [Bassetto and Cui \(2018\)](#) study how that backing works when interest rates are low.

⁵For example, [SIX \(2022\)](#) describes ongoing collateral transfers and multiple daily valuation checks in its collateral management service. The fee schedule in [LCH Limited \(2025\)](#) distinguishes minimum monthly charges from one-time onboarding fees.

and $\lim_{c \rightarrow \infty} F(c) = 1$. Its density f is positive and continuously differentiable on $(0, \infty)$, extends continuously to zero, and $f(0) = 0$. There is a mode $c_m > 0$ such that

$$f'(c) > 0 \quad \text{for } 0 < c < c_m, \quad f'(c) < 0 \quad \text{for } c > c_m. \quad (3)$$

Single-peakedness places a shape restriction on the cost density without choosing a parametric family. Common distributions satisfying Assumption 1 include the log-normal and, when the shape parameter exceeds one, the Gamma, Weibull, and log-logistic distributions. Appendix B relaxes the condition $f(0) = 0$, which admits distributions such as the normal truncated at zero.⁶

Write F^{-1} for the inverse cost distribution, with $F^{-1}(0) = 0$. For a share A , the value $F^{-1}(A)$ is the cost of the marginal institution when adoption is assigned to the lowest-cost users. Define

$$\mathcal{C}(A) = \int_0^A F^{-1}(u) \, du = \int_0^{F^{-1}(A)} c \, dF(c), \quad 0 \leq A < 1. \quad (4)$$

This is the lowest aggregate implementation cost of an adoption share A , obtained by assigning adoption to the lowest-cost institutions. At $A = 1$, use the extended value $\mathcal{C}(1) = \lim_{A \uparrow 1} \mathcal{C}(A)$, possibly $+\infty$.

Definition 1 (Adoption equilibrium). An adoption equilibrium at strength b is a share $A \in [0, 1]$ satisfying

$$A = F(bA). \quad (5)$$

Institutions with $c < bA$ adopt and those with $c > bA$ do not; the indifferent type has zero mass.

3 Adoption Equilibria and Potential

Zero adoption is always an equilibrium. If no institution uses the government claim as its reference, an individual institution obtains no network benefit from adopting but still incurs its implementation cost, so it has no incentive to adopt on its own. The more interesting question is whether the model also admits nontrivial equilibria with positive adoption. When do such equilibria exist, and if they exist, how many can there be?

To answer these questions, consider a positive equilibrium with a marginal adopter of cost c . Since the marginal adopter satisfies $c = bA$ and the adoption share is $A = F(c)$, the benchmark strength needed to sustain a cutoff c is $\frac{c}{F(c)}$. Define the lowest such strength by

$$b_f \equiv \min_{c > 0} \frac{c}{F(c)}, \quad c_f \equiv \arg \min_{c > 0} \frac{c}{F(c)}, \quad A_f \equiv F(c_f). \quad (6)$$

⁶The existence and selection thresholds keep their order, and the results on corridors, transfers, and prices go through. The main change is that once $b \geq 1/f(0)$, any positive initial share grows under noiseless renewal.

The next theorem shows that, under Assumption 1, these objects are well defined and that b_f completely characterizes the existence and multiplicity of positive-adoption equilibria.

Theorem 1 (Equilibrium existence and multiplicity). *Under Assumption 1, strictly positive equilibria exist if and only if $b \geq b_f$. At $b = b_f$, the unique positive equilibrium is A_f . If $b > b_f$, there are exactly two positive equilibria $A_-(b) < A_f < A_+(b)$.*

The existence threshold measures how useful the reference must be before any group of institutions can sustain common use. A small group offers few compatible counterparties, so even low-cost institutions have little reason to join. A larger group offers more benefits, but sustaining it requires participation by institutions with higher implementation costs. Below the threshold, no positive network size balances these two forces.

Above the threshold, the feedback from common use is strong enough to support both a smaller and a larger network. In each equilibrium, the benefit available at that network size covers the costs of precisely those institutions using it. Neither equilibrium removes the alternative of staying out together. The theorem therefore establishes the possibility of adoption, rather than predicting that institutions must coordinate on it.

Potential geometry. For a cutoff allocation with adoption share A , define the potential

$$\mathcal{P}(A; b) = \frac{b}{2}A^2 - \mathcal{C}(A), \quad 0 \leq A \leq 1. \quad (7)$$

For a given adoption share, assigning adoption to the lowest-cost institutions minimizes implementation costs. The derivative has a direct incentive interpretation:

$$\mathcal{P}_A(A; b) = bA - F^{-1}(A). \quad (8)$$

Increasing the cutoff slightly brings in an institution with cost $F^{-1}(A)$. The derivative is its private adoption gain at the prevailing network size. Thus the positive stationary points of $\mathcal{P}$, i.e., $\mathcal{P}_A(A; b) = 0$, are exactly the positive adoption equilibria.

Zero is a strict one-sided local potential maximum for every finite b : near zero, the marginal cost $F^{-1}(A)$ exceeds bA . When $b > b_f$, the lower root A_- is a strict local minimum and the upper root A_+ is a strict local maximum. At $b = b_f$, the two positive stationary points merge at A_f and disappear when b falls further. This meeting point is the *fold*. The potential is decreasing on either side of it, so the fold itself is neither a local maximum nor a local minimum. When $b \leq b_f$, zero is the unique global potential maximizer. (See the proof of Theorem 1 for a formal argument.)

Figure 1 illustrates the equilibrium branches and their potential geometry. Panel (b) distinguishes two positive stationary points above b_f from one at b_f and none below it. At the threshold, the potential has a horizontal tangent at A_f but continues to decrease on both sides; this point lies strictly below the value at nonadoption. The local stability labels in panel (a) are established in Proposition 1. The marker b_p previews the selection threshold characterized in Theorem 3.

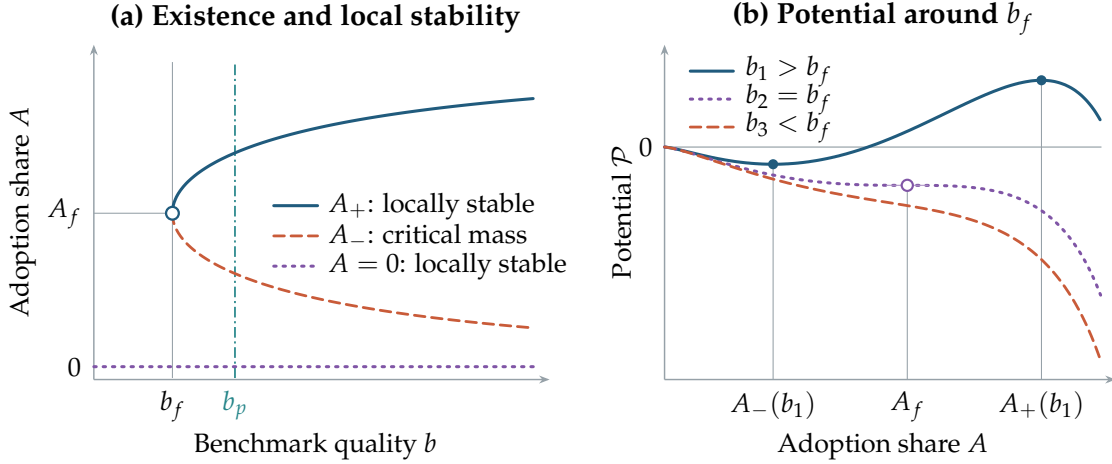


Figure 1: Equilibrium branches and potential geometry. Panel (a) shows the positive branches meeting at b_f and previews the selection threshold b_p . The stability labels concern the branches away from the fold. In panel (b), $b_1 > b_f$ gives a local minimum and a local maximum. At $b_2 = b_f$, the potential remains decreasing, with a single positive stationary point at A_f below zero. For $b_3 < b_f$, there is no positive stationary point. The diagrams are qualitative; the comparison values b_i are specific to this figure.

We define the set of potential maximizers

$$\mathcal{M}(b) = \arg \max_{0 \leq A \leq 1} \mathcal{P}(A; b). \quad (9)$$

This set is nonempty and compact. For $b > b_f$, the only global candidates are 0 and $A_+(b)$. Local geometry alone does not rank these two candidates. The next section first explains why their global potential ranking matters for behavior, then pins down the potential maximizer.

4 Adjustment and Long-Run Selection

The equilibrium equation describes which adoption shares are self-consistent, but it does not describe how institutions revise their decisions. We first introduce the individual logit choice rule. We then apply it to the continuum of institutions to study renewal from a given initial user base. Finally, we introduce a finite population to study the stationary distribution of noisy revisions. The same payoff comparison and revision opportunities govern both population

descriptions. Benchmark strength b and implementation costs remain fixed throughout this section.

Each institution receives revision opportunities according to an independent Poisson clock with rate one. At an opportunity, it compares the current payoff from adoption with the payoff from nonadoption, taking the other institutions' current choices as given. Its chosen action remains in place until its next revision opportunity.

Let g denote the payoff gain from adoption rather than nonadoption. The institution chooses adoption with probability

$$L(g; \eta) = \frac{\exp(g/\eta)}{1 + \exp(g/\eta)}, \quad \eta > 0. \quad (10)$$

It chooses nonadoption with the complementary probability. The parameter η measures revision noise. The more profitable action is more likely, but both actions have positive probability at any finite payoff difference. A revision can therefore change an institution's action or leave it unchanged.

As η vanishes, the rule approaches best response: adoption is chosen when $g > 0$, nonadoption when $g < 0$, and the two actions have equal probability at exact indifference. For a cost- c institution in the continuum economy, the payoff gain is $g = bA - c$.

4.1 Continuum Renewal and Critical Mass

Fix $\eta > 0$. Let $a(c, t; \eta)$ be the fraction of cost- c institutions that adopt at time t , and let

$$A(t; \eta) = \int_0^\infty a(c, t; \eta) dF(c)$$

be aggregate adoption. Both quantities are scalar shares; the parameter after the semicolon records revision noise. Take an initial allocation with aggregate share $A(0; \eta) = A_0 \in [0, 1]$.

A cost- c institution adopts with probability $L(bA(t; \eta) - c; \eta)$ whenever it revises. At unit-rate revision, the inflow from current nonadopters is

$$[1 - a(c, t; \eta)]L(bA(t; \eta) - c; \eta),$$

while the outflow from current adopters is

$$a(c, t; \eta)[1 - L(bA(t; \eta) - c; \eta)].$$

Subtracting outflows from inflows gives

$$\partial_t a(c, t; \eta) = L(bA(t; \eta) - c; \eta) - a(c, t; \eta).$$

Integrating these type-level flows over costs yields

$$\dot{A}(t; \eta) = \int_0^\infty L(bA(t; \eta) - c; \eta) dF(c) - A(t; \eta). \quad (11)$$

The integral is the share that would choose adoption at a revision opportunity, given the current network. Actual adoption moves toward that share as institutions reconsider their decisions. In this continuum description, probabilistic individual choices enter through deterministic adoption flows, so aggregate adoption follows a deterministic path from its initial share.⁷

As $\eta \downarrow 0$, institutions choose the more profitable action. Since F is continuous, the cost type exactly indifferent at a given adoption share has zero mass. The aggregate best response therefore converges to $F(bA)$. Writing $A(t)$ for noiseless adoption gives

$$\dot{A}(t) = F(bA(t)) - A(t), \quad A(0) = A_0 \in [0, 1]. \quad (12)$$

Equivalently, the noiseless type fractions satisfy $\partial_t a(c, t) = \mathbf{1}\{c < bA(t)\} - a(c, t)$ almost everywhere. The revision opportunities and payoff comparison are unchanged; only the noise in the choice at an opportunity has disappeared. We now ask where this renewal process leads from a given initial installed base. Note that equilibria are exactly the fixed points of this process.

Proposition 1 (Noiseless adjustment dynamics). *Under Assumption 1, zero is locally asymptotically stable under (12) for every finite b . If $b < b_f$, every initial share converges to zero. If $b > b_f$,*

$$\begin{aligned} A_0 \in [0, A_-(b)) &\implies A(t) \longrightarrow 0, \\ A_0 \in (A_-(b), 1] &\implies A(t) \longrightarrow A_+(b). \end{aligned}$$

The lower root is unstable. At $b = b_f$, initial shares below A_f converge to zero and those at or above A_f converge to A_f .

The lower equilibrium is a critical mass because an institution's reason to use the reference depends on how many counterparties already use it. Below that share, the available benefits attract fewer users than the network currently has, so adoption declines and reinforces further exit. Above it, the network moves toward the high-adoption equilibrium. The same benchmark quality can therefore sustain an established user base while failing to support a smaller one. This

⁷The derivation does not require the current adopters to be the lowest-cost institutions.

dependence on the installed base follows from the adoption incentives, even though institutions face no separate switching cost. These basins explain the stability labels in Figure 1(a).

Corollary 1 (Launching from an initial share). *For $A_0 \in (0, A_f)$, convergence under (12) to the high equilibrium occurs if and only if*

$$b > b_{\text{initial share}}(A_0) \equiv \frac{F^{-1}(A_0)}{A_0} > b_f. \quad (13)$$

The strength threshold decreases strictly with A_0 and diverges as $A_0 \downarrow 0$.

Launching a reference and maintaining one impose different requirements. A small initial group offers few compatible counterparties, so the claim must be especially useful to attract enough additional institutions. A larger initial group supplies more of that benefit itself and can grow at a lower benchmark strength. The corollary states the precise trade-off between initial participation and the quality needed for eventual convergence to the high-adoption equilibrium.

In particular, the existence of a positive equilibrium does not guarantee that an arbitrarily small launch will reach it. With no revision noise, zero adoption cannot restart on its own, and even a small positive base may continue to contract. These deterministic basins describe adjustment from a given initial share. To study long-run selection when individual choices remain noisy, we next apply the same revision rule to a finite population.

4.2 Finite-Population Revision and Stationary Selection

Stationary selection concerns the long-run frequencies of adoption configurations when institutions continue to revise with positive noise. For this question, we now introduce a finite population. This preserves random fluctuations in aggregate adoption and makes the process an irreducible Markov chain.

Consider a finite population setting with n institutions. For $n \geq 2$, let

$$\mathbf{a} = (a_1, \dots, a_n) \in \{0, 1\}^n$$

be the profile of all institutions' adoption actions. The vector $\mathbf{a}_{-i}$ collects the actions other than i 's. Individual actions a_i remain scalar. Choose a representative, deterministic cost array and define aggregate adoption by

$$c_{i,n} = F^{-1}\left(\frac{i}{n+1}\right), \quad i = 1, \dots, n, \quad A_n(\mathbf{a}) = \frac{1}{n} \sum_{i=1}^n a_i. \quad (14)$$

The empirical cost distribution converges to F as n grows; costs do not change at revision dates.

Institution i 's gain from choosing adoption rather than nonadoption is

$$g_{i,n}(\mathbf{a}_{-i}) = \frac{b}{n} \sum_{j \neq i} a_j - c_{i,n}. \quad (15)$$

Excluding its own action makes the comparison depend only on the other institutions' choices. Its difference from $bA_n - c_{i,n}$ is at most b/n and disappears in the large-population limit.

The revision process and its potential. Each institution uses the unit-rate Poisson revision clock and logit rule 10. At a revision opportunity, institution i chooses adoption with probability $L(g_{i,n}(\mathbf{a}_{-i}); \eta)$ and nonadoption with the complementary probability. Revision clocks and choice draws are independent across institutions conditional on the current profile. The state records the individual actions, not only the aggregate share, because institutions have different fixed costs.

The revision process defines a continuous-time Markov chain on the set of adoption profiles $\{0, 1\}^n$. Total revision opportunities arrive at rate n ; conditional on an opportunity, the revising institution is uniform over the population. The chain observed at those opportunities is therefore the usual one-at-a-time logit revision chain. The continuous-time process and this embedded chain have the same invariant distribution.

The finite game has the exact potential

$$\Phi_n(\mathbf{a}; b) = \frac{b}{n} \sum_{i < j} a_i a_j - \sum_{i=1}^n c_{i,n} a_i. \quad (16)$$

When one institution switches from nonadoption to adoption, the increase in Φ_n is exactly its payoff gain in (15). For a fixed number of adopters, the first term is unchanged across profiles, so the highest potential is obtained by selecting the lowest-cost institutions. Normalizing by n and increasing the population then gives the cutoff potential $\mathcal{P}(A; b)$ in (7). The approximation is established in the proof of Theorem 2.

The invariant distribution. For fixed n and $\eta > 0$, every profile can be reached through a sequence of revisions with positive probability. The finite continuous-time chain is therefore irreducible and has a unique invariant distribution. This distribution describes the long-run frequencies of adoption profiles.

The invariant distribution can be obtained directly from the logit revision rule. Let $q_{n,\eta}(\mathbf{a}, \mathbf{a}')$ denote the transition rate from profile $\mathbf{a}$ to a distinct profile $\mathbf{a}'$. A sufficient condition for a

probability distribution μ to be invariant is *detailed balance*:

$$\mu(\mathbf{a})q_{n,\eta}(\mathbf{a}, \mathbf{a}') = \mu(\mathbf{a}')q_{n,\eta}(\mathbf{a}', \mathbf{a}) \quad \text{for all distinct } \mathbf{a}, \mathbf{a}'.$$

The left-hand side is the probability flow from $\mathbf{a}$ to $\mathbf{a}'$, and the right-hand side is the reverse flow. Detailed balance equates these flows pair by pair; summing over the other profiles then equates the total inflow and outflow at each profile, so the distribution remains unchanged over time.

To verify this condition, consider neighboring profiles $\mathbf{a}$ and $\mathbf{a}'$ that differ only because institution i switches from nonadoption to adoption. The reverse revision faces the same actions $\mathbf{a}_{-i}$ of the other institutions. Since the individual revision rate is one, the logit rule gives

$$\frac{q_{n,\eta}(\mathbf{a}, \mathbf{a}')}{q_{n,\eta}(\mathbf{a}', \mathbf{a})} = \frac{L(g_{i,n}(\mathbf{a}_{-i}); \eta)}{1 - L(g_{i,n}(\mathbf{a}_{-i}); \eta)} = \exp\left\{\frac{\Phi_n(\mathbf{a}'; b) - \Phi_n(\mathbf{a}; b)}{\eta}\right\}.$$

The last equality uses the exact-potential property: the change in Φ_n equals the revising institution's payoff gain. Thus assigning probabilities proportional to $\exp\{\Phi_n/\eta\}$ satisfies detailed balance. Profiles that differ in more than one action have zero direct transition rates in both directions. Consequently, the unique invariant distribution is

$$\mu_{n,\eta}(\mathbf{a}) = \frac{\exp\{\Phi_n(\mathbf{a}; b)/\eta\}}{\sum_{\tilde{\mathbf{a}} \in \{0,1\}^n} \exp\{\Phi_n(\tilde{\mathbf{a}}; b)/\eta\}}. \quad (17)$$

The next theorem establishes which aggregate adoption levels this distribution selects as the population grows and revision noise vanishes. For a nonempty set $S \subseteq [0, 1]$, write $\text{dist}(A, S) = \inf_{y \in S} |A - y|$.

Theorem 2 (Stationary logit selection). *Suppose Assumption 1 holds and fix $b > 0$. For every sequence $\eta_n > 0$ with $\eta_n \rightarrow 0$ and every $\delta > 0$,*

$$\mu_{n,\eta_n}(\text{dist}(A_n(\mathbf{a}), \mathcal{M}(b)) \geq \delta) \rightarrow 0 \quad \text{as } n \rightarrow \infty. \quad (18)$$

Thus stationary adoption concentrates on the set of global potential maximizers. In particular, whenever $\mathcal{M}(b) = \{A^*(b)\}$, stationary adoption converges in probability to the unique potential maximizer:

$$A_n(\mathbf{a}) \xrightarrow{\mathbb{P}} A^*(b), \quad \mathbf{a} \sim \mu_{n,\eta_n}.$$

The theorem explains why individual revisions can select one equilibrium without any institution choosing the outcome for the network as a whole. Each revising institution responds only to its own gain. The exact-potential property translates those gains into the relative

probability flows between adoption profiles, and the Gibbs measure ranks their long-run frequencies accordingly. As noise becomes small and the population grows, stationary adoption is concentrated near the global potential maximizers. When there is only one such maximizer, almost all stationary probability is assigned to adoption shares near that outcome; when several tie, the theorem does not choose their limiting weights.

This criterion is stronger than resistance to a small loss of users. A high-adoption network can be locally persistent and still lose the long-run comparison with nonadoption. Nor does the criterion simply favor the network with more participants: the costs of those participants enter the potential as well. The proof accounts for all possible assignments of adoption across institutions, including profiles in which some high-cost institutions adopt and some low-cost institutions do not. It does not assume that revisions remain within cost-ordered profiles.

The same proof also yields convergence of expected normalized potential:

$$\mathbb{E}_{\mu_{n,\eta}} \left[\frac{\Phi_n(\mathbf{a}; b)}{n} \right] \longrightarrow \max_{0 \leq A \leq 1} \mathcal{P}(A; b).$$

These are stationary selection results: the long-run distribution is taken at each finite n and positive η before the large-population, vanishing-noise limit. They do not assert that a network starting from a particular installed base immediately moves to the potential-maximizing equilibrium.

Why the order of limits matters. Section 4.1 starts with a continuum, derives its adoption flows, and studies the noiseless path as time passes. The finite model introduced here also provides a precise connection to that description. Lemma 2 in Appendix A.4 shows that, on any fixed interval $[0, T]$, its aggregate path approaches (11) as the population grows at fixed positive noise. If the population grows and noise vanishes jointly, the limiting path is (12), provided initial aggregate adoption converges to A_0 . The initial adopters need not be ordered by cost.

This gives a finite-population interpretation of deterministic adjustment. First take a large population and vanishing noise while keeping the observation horizon T fixed. The resulting approximation removes aggregate fluctuations and gives the noiseless renewal equation. Only then study the solution of that equation as $t \rightarrow \infty$. Proposition 1 shows that its destination depends on the initial installed base. A close approximation on each fixed time interval does not guarantee closeness over arbitrarily long periods at a given finite population size and positive noise level.

Stationary selection takes time to infinity first. Hold n finite and $\eta > 0$, and let the distribution of the action profile converge to $\mu_{n,\eta}$. Only afterward let the population grow and noise vanish, as in Theorem 2. In this order, the finite system has time to move between adoption configurations

before its fluctuations disappear. The limiting distribution therefore need not select the same outcome as noiseless adjustment from a particular initial share.

Keeping $\eta > 0$ in the continuum equation does not reverse this order. That equation already replaces individual random choices by aggregate flows. Given an initial share, its path is deterministic, and a path at a steady state has no random aggregate disturbance to move it away. Following that equation for arbitrarily long therefore does not compute the invariant distribution of the finite system. Local persistence and stationary selection concern different long-run comparisons under the same individual revision rule.

4.3 The Equilibrium-Selection Threshold

The revision mechanism provides a reason to compare global potentials. It remains to determine when positive adoption wins that comparison. Zero has potential zero. For $b > b_f$, the only competing global candidate is $A_+(b)$, so selection reduces to the sign of $\mathcal{P}(A_+(b); b)$.

Theorem 3 (Equilibrium selection). *Under Assumption 1, there is a unique finite threshold b_p and a positive share $A_p = A_+(b_p)$ such that*

$$\mathcal{M}(b) = \begin{cases} \{0\}, & b < b_p, \\ \{0, A_p\}, & b = b_p, \\ \{A_+(b)\}, & b > b_p. \end{cases} \quad (19)$$

The threshold is characterized by

$$b_p = \min_{0 < A < 1} \frac{2\mathcal{C}(A)}{A^2}, \quad (20)$$

with the minimum attained uniquely at A_p .

Combining Theorems 2 and 3, in the large-population, vanishing-noise limit, stationary adoption converges in probability to zero for $b < b_p$ and to $A_+(b)$ for $b > b_p$.⁸ Figure 2 shows the global comparison behind this selection. The upper local maximum is above zero when the illustrated strength exceeds b_p , touches zero at b_p , and lies below zero for strengths below b_p .

Existence and selection therefore impose different requirements on benchmark quality. The threshold b_f is the lowest quality at which a positive network can sustain itself. The threshold b_p is the quality at which the high-adoption equilibrium first matches nonadoption in potential. These thresholds satisfy⁹

$$b_f < b_p < 2b_f, \quad A_p > A_f. \quad (21)$$

⁸At $b = b_p$, stationary adoption concentrates on $\{0, A_p\}$, but the result does not specify the limiting probability assigned to each outcome.

⁹The proof of Theorem 3 establishes the bounds in (21).

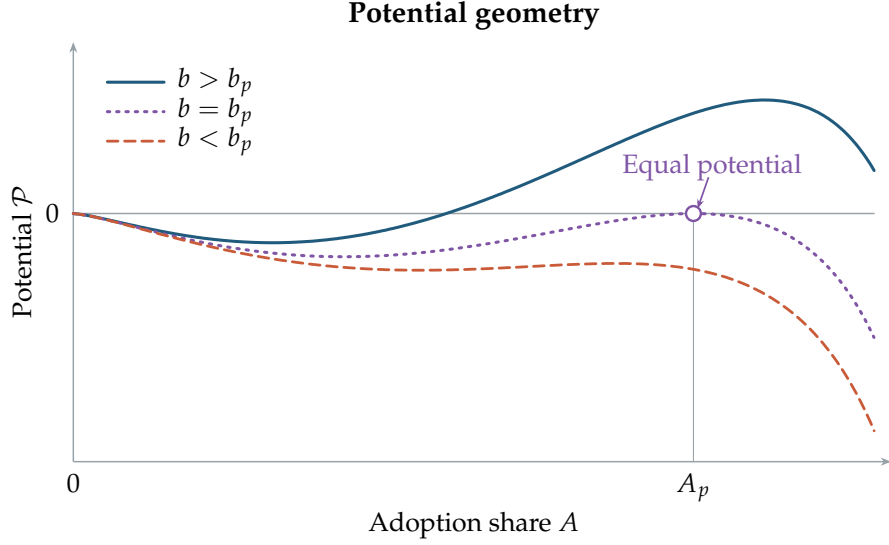


Figure 2: Potential comparison and stationary selection. The positive local maximum is above zero for $b > b_p$, equal to zero at A_p for $b = b_p$, and below zero for the illustrated $b_f < b < b_p$. Since nonadoption always has zero potential, these cases select the high-adoption equilibrium, permit both 0 and A_p , and select nonadoption, respectively.

Thus making a positive network feasible is not enough to make it the long-run selected outcome. The bound holds for every distribution satisfying Assumption 1: selection never requires more than twice the quality needed for existence.

The gap arises because the incentives supporting an operating network are stronger than those supporting participation at smaller network sizes. At b_f , the network with adoption share A_f generates just enough benefit to cover the cost of its marginal user. At every smaller positive share, however, the network benefit falls short of the cost of the marginal institution needed to sustain that share. The potential records these private marginal incentives across network sizes. Their negative values below A_f leave the positive equilibrium with lower potential than nonadoption at the point where it first becomes feasible.

Higher quality strengthens the incentives to use the reference without changing implementation costs. It therefore improves the potential of adoption relative to nonadoption. The theorem shows that this ranking reverses at a single threshold: at b_p , the high-adoption equilibrium and nonadoption have equal potential; above b_p , the high-adoption equilibrium wins the comparison. The interval (b_f, b_p) consequently consists of qualities at which a positive network can sustain itself but is not selected by the stationary logit criterion.

5 Debt Corridors and Corrective Policy

We now examine how the supply of eligible government debt affects its usefulness as a common reference. Benchmark quality combines two components. Effective market depth $m(x)$ captures

how readily institutions can obtain and trade the claim, given the infrastructure that supports its use. We take it to be increasing in the face value of eligible debt x : a larger supply makes the claim more widely available and supports trading in it. [Fleming \(2002\)](#) finds that larger Treasury issues are more liquid. Payoff retention $s(x; \varphi) \in (0, 1]$ is the fraction of the promised real payment delivered to a holder. Holding the backing resources φ fixed, we take retention to be weakly decreasing in x , since additional issuance spreads those resources across more promised payments. A low interest rate relative to growth lowers the fiscal cost of that stock ([Blanchard, 2019](#); [Reis, 2022](#)), but it does not undo the dilution. Both components are continuous, and their product determines benchmark quality:

$$b(x; \varphi) = m(x)s(x; \varphi).$$

These two components create a trade-off between availability and payoff dilution. When the debt stock is small relative to its backing, retention can be close to one, but limited supply makes the claim difficult to use as a common reference. Additional issuance then primarily improves market depth. At high debt levels, the market is already well supplied, so further issuance contributes little additional depth. Retention, however, continues to decline as the same backing supports more claims. The loss in retained payoff can then outweigh the remaining gains from a deeper market. The schedule is hump-shaped for a different reason from the debt Laffer curve in [Brunnermeier et al. \(2024\)](#) and the revenue-maximizing debt level in [Bayer et al. \(2023\)](#). In those papers the liquidity value per unit falls as more debt is issued. Here depth rises with supply while backing per claim falls.

This trade-off motivates a quality schedule that first rises and then falls with debt supply. A very small issue provides too little depth, while a sufficiently large issue suffers enough payoff dilution to undermine its usefulness. Rather than specify a particular functional form for either component, we impose the following shape restriction on their product.

Assumption 2 (Single-peaked quality). *For each $\varphi > 0$, $b(\cdot; \varphi)$ is positive and continuous on $(0, \infty)$, tends to zero at both endpoints, and is strictly increasing up to a unique $\hat{x}(\varphi)$ and strictly decreasing thereafter.*

The peak $\hat{x}(\varphi)$ separates the range in which the benefits of greater depth dominate from the range in which payoff dilution dominates. Comparing $b(x; \varphi)$ with the adoption thresholds b_f and b_p therefore translates the existence and selection requirements into restrictions on debt supply. We do not solve the issuance problem of [Bassetto and Cui \(2024\)](#), or the choice of maturity in [Jiang et al. \(2026\)](#). The model has a single eligible claim, and we take its stock as given.

5.1 Feasibility and Coordination Corridors

We now map the two private-adoption thresholds back into debt supply. For any strength requirement $\tau > 0$, define

$$\mathcal{X}(\tau) = \{x > 0 : b(x; \varphi) \geq \tau\}, \quad b_{\max} = b(\hat{x}(\varphi); \varphi). \quad (22)$$

Write $\mathcal{X}_f = \mathcal{X}(b_f)$ for the feasibility corridor and $\mathcal{X}_p = \mathcal{X}(b_p)$ for the coordination corridor. The first records where a positive private equilibrium exists; the second records where a positive equilibrium is a global potential maximizer.

Corollary 2 (Corridors and comparative statics). *Under Assumptions 1 and 2, $\mathcal{X}_p \subseteq \mathcal{X}_f$. If $b_{\max} > b_p$, both corridors are nondegenerate and the inclusion is strict. If $b_f < b_{\max} < b_p$, the feasibility corridor is nondegenerate but the coordination corridor is empty.*

If another admissible distribution $\tilde{F}$ satisfies $\tilde{F}(c) \geq F(c)$ for all c , both thresholds are weakly lower and both corridors weakly larger. Multiplying every cost by $\kappa > 0$ multiplies b_f, b_p by κ and leaves A_f, A_p unchanged.

The two ends of each corridor reflect different reasons a reference can fail to attract users. With too little eligible debt, the market is too thin for common use to be worthwhile. With too much, the claim's retained payoff is diluted enough to outweigh the improvement in depth. A positive network is possible only between these two extremes. Stationary selection requires better quality than existence alone, so it moves both boundaries inward.

Implementation costs matter at both stages. Lower costs make it easier for a group of users to sustain its participation and improve the potential of adoption relative to staying out. Both corridors therefore expand when costs fall in the stated distributional sense, holding debt quality fixed. Market infrastructure that lowers the recurring cost of using the claim widens both corridors, with debt and backing unchanged. A common rescaling of all costs changes how strong the benchmark must be, but not the critical adoption shares: it changes the level of the cost schedule without changing its relative shape.

Initial adoption creates a separate launch requirement. For $A_0 \in (0, A_f)$, the set of debt levels from which noiseless adoption converges to the high equilibrium is

$$\{x : b(x; \varphi) > b_{\text{initial share}}(A_0)\}.$$

It is a nonempty open interval exactly when $b_{\max} > b_{\text{initial share}}(A_0)$, by Corollary 1.

5.2 Internalizing the Adoption Externality

The analysis so far concerns private equilibria and their selection. We now hold x , φ , and hence b fixed and ask what an adoption intervention can accomplish. This requires an explicit measure of the benefits and costs of adoption, including allocations that need not be private equilibria without the intervention.

Network surplus. At a cost-ordered allocation of size A , each adopter obtains gross benefit bA , so aggregate net adoption surplus is

$$U(A; b) = bA^2 - C(A). \quad (23)$$

For any fixed A , assigning adoption to the lowest-cost institutions maximizes this surplus. Thus maximizing (23) over A is also the optimum over all assignments of adoption to cost types. The objective counts the institutions' network benefits and real implementation costs; it holds debt and backing fixed and includes no additional fiscal distortions or investor services.

At an interior cutoff allocation, the private gain of the marginal adopter is $bA - F^{-1}(A)$, whereas the marginal change in aggregate surplus is $U_A(A; b) = 2bA - F^{-1}(A)$. The difference, bA , is the positive externality that the additional adopter creates for institutions already using the reference. This distinction also explains why a global potential comparison need not coincide with a surplus comparison. The potential measures marginal private incentives, not the sum of all adopters' payoffs.

Since $U(A; b) = \mathcal{P}(A; 2b)$, Theorem 3 immediately gives the planner's allocation and the threshold for positive net gains:

$$b_w \equiv \min_{0 < A < 1} \frac{C(A)}{A^2} = \frac{b_p}{2} < b_f, \quad (24)$$

$$\arg \max_{0 \leq A \leq 1} U(A; b) = \mathcal{M}(2b).$$

A positive allocation with strictly positive net surplus exists exactly when $b > b_w$. At $b = b_w$, both zero and A_p attain surplus zero; above it, the surplus-maximizing share is $A_+(2b)$. The interval (b_w, b_f) is therefore relevant to the intervention problem: there are positive-surplus allocations there, but no positive *unsubsidized* adoption equilibrium.

A matching transfer. Consider a policy paying each adopter

$$\tau_\lambda(A) = \lambda bA, \quad 0 \leq \lambda \leq 1. \quad (25)$$

Transfers are financed by nondistortionary lump-sum taxes on a passive outside sector with sufficient resources. They are not drawn from the backing resources φ . Including taxpayers, transfers cancel and aggregate net surplus remains $U(A; b)$. The finance assumption separates an adoption intervention from a change in benchmark quality.

The subsidized private gain is $(1 + \lambda)bA - c$, and the induced potential is

$$\mathcal{P}_\lambda(A; b) = \frac{(1 + \lambda)b}{2} A^2 - \mathcal{C}(A). \quad (26)$$

In the finite game, adopter i receives $\lambda(b/n) \sum_{j \neq i} a_j$, with taxes paid by the outside sector. The revision protocol is unchanged; its gains and potential replace b by $(1 + \lambda)b$. A policy therefore changes the incentives within the same selection mechanism.

Proposition 2 (Corrective transfers and implementation). *Under Assumption 1 and (25):*

- (i) *Positive private equilibria exist if and only if $(1 + \lambda)b \geq b_f$. A positive potential maximizer exists if and only if $(1 + \lambda)b \geq b_p$; it is the unique global maximizer when the latter inequality is strict.*
- (ii) *At $\lambda = 1$, $\mathcal{P}_1(A; b) = U(A; b)$ for every A . Stationary small-noise adoption converges to the unique surplus-maximizing share when $b \neq b_w$ and concentrates on $\{0, A_p\}$ when $b = b_w$.*

This matching transfer specializes the Pigouvian stochastic implementation approach of [Sandholm \(2007\)](#) to the adoption game. The transfer rewards a benefit that an institution otherwise ignores. Joining the reference helps not only the new adopter but also the users who can now interact with it. At full correction, the payment equals that additional benefit to existing users. Private adoption decisions then reflect the marginal change in total network surplus. Because the revision protocol selects according to the induced potential, aligning that potential with surplus also aligns long-run selection with the planner's adoption objective. When quality is high enough, even a rate below full correction can make positive adoption a potential maximizer. The transfer changes the relative attractiveness of using the reference and staying out.

Launching versus maintaining a network. A matching transfer can help launch a network without being needed permanently. Fix $b > b_f$ and consider noiseless renewal from an initial share $A_0 \in (0, A_-(b))$. Without intervention, this initial network would shrink toward zero. Suppose a matching rate $\lambda \in (0, 1]$ satisfies

$$b > \frac{F^{-1}(A_0)}{(1 + \lambda)A_0}. \quad (27)$$

By Corollary 1, this condition places the initial share in the basin of the high-adoption equilibrium at effective strength $(1 + \lambda)b$.

While the transfer remains in place, adoption increases toward $A_+((1 + \lambda)b)$. Since this subsidized equilibrium lies above $A_f > A_-(b)$, the adoption share exceeds the original critical mass $A_-(b)$ after a finite time. The government need not wait until the subsidized equilibrium is reached. It can withdraw the transfer at any finite date T at which

$$A(T) > A_-(b).$$

Thereafter, adoption follows the unsubsidized renewal law $\dot{A}(t) = F(bA(t)) - A(t)$. Proposition 1 therefore implies

$$\lim_{t \rightarrow \infty} A(t) = A_+(b),$$

without further transfers.

This distinguishes launching a network from maintaining it. A small initial network offers too few compatible counterparties to attract additional users on its own. The transfer temporarily supplements those benefits and helps build the installed base. Once participation exceeds the unsubsidized critical mass, the network's own benefits support convergence to the high-adoption equilibrium. Temporary support can therefore change which equilibrium is reached without changing the equilibrium that ultimately sustains the network.

5.3 A Conditional Pricing Implication

As a separate pricing extension, suppose passive risk-neutral investors discount at $\beta \in (0, 1)$. Each unit of the claim delivers the retained real payoff $s(x; \varphi)$ and a next-period per-unit service hA , where $h > 0$. Its price is therefore

$$p(x, A) = \beta\{s(x; \varphi) + hA\}. \quad (28)$$

This extension takes adoption as determined by the unsubsidized game. The additional investor service is not included in the adoption objective of Section 5.2; the equation values the network conditional on adoption rather than changing institutions' revision incentives. Equation (28) is the cash-flow plus service-flow decomposition of Brunnermeier et al. (2024), with the service proportional to the adoption share. The adoption game determines the quantity of service A ; the parameter h prices each unit and is taken as given. Write $b(x) = b(x; \varphi)$ at fixed φ .

Let $\mathcal{X}(b_f) = [x_L(b_f), x_U(b_f)]$. We consider an operating network continued along the high-adoption branch up to $x_U(b_f)$. At that boundary its adoption share approaches A_f . Immediately beyond the boundary, no positive adoption equilibrium exists.

Corollary 3 (Loss of convenience value at the fold). *Under Assumptions 1 and 2, suppose $b_{\max} > b_f$. The branch-continuation price loss at the upper fold is*

$$\lim_{x \uparrow x_U(b_f)} p(x, A_+(b(x))) - \lim_{x \downarrow x_U(b_f)} p(x, 0) = \beta h A_f. \quad (29)$$

The loss comes from the disappearance of common use, rather than a sudden reduction in the claim's retained cash payment. Along the operating branch, investors value the services supplied by a positive network. Once the claim no longer supports any positive equilibrium, those services have no positive-equilibrium counterpart. Because payoff retention is continuous at the boundary, the difference between the limiting prices is exactly the discounted value of the lost service. [He et al. \(2022\)](#) explain the loss of Treasury convenience yields in March 2020 by dealer balance-sheet constraints. The fold here is a different mechanism, but it shares the feature that the service part of the price can disappear while the cash payoff does not change.

6 Conclusion

The limits of private coordination depend on whether the obstacle is insufficient participation or insufficient benchmark quality. A claim may support a functioning network without attracting enough initial users to establish it. Conversely, a large installed base cannot sustain an unsubsidized equilibrium once quality falls below the existence threshold. The stationary logit comparison imposes a further requirement on selection among feasible outcomes, but does not change this underlying feasibility constraint.

This distinction gives temporary support a specific role. When a self-sustaining equilibrium exists, a matching transfer that satisfies the launch condition can help institutions build a sufficient user base. Under noiseless renewal at fixed quality, support can then be withdrawn once adoption exceeds the original critical mass. The intervention changes which equilibrium is reached, rather than the unsubsidized equilibrium itself. Successful launch therefore does not remove the constraints that debt supply and backing place on continued use. The same launch problem applies to a proposed safe asset that does not yet have users, such as the senior tranche of pooled euro-area sovereign bonds in [Brunnermeier et al. \(2017\)](#). Joint bonds issued by the European Union since 2020 face the same problem as a new issuer.

The analysis isolates these adoption constraints rather than solving the issuer's financing problem. Their relevance extends beyond participation: the price comparison at the existence boundary reflects a loss of common-use services even though the retained cash payoff is continuous. A liquidity premium of the kind assumed in models of optimal public debt is available only inside the feasibility corridor, and it is the long-run outcome only inside the narrower

coordination corridor.

A Proofs

Throughout the appendix, write $Q(u) = F^{-1}(u)$ for $0 \leq u < 1$, with $Q(0) = 0$, and let $R(c) = c/F(c)$ for $c > 0$. Both shorthand functions are used only in the proofs.

A.1 Single-Peaked Costs, Equilibria, and Potential Geometry

Lemma 1 (Single-peaked density implies single crossing). *Under Assumption 1, there is a unique $c_f > c_m$ such that $c_f f(c_f) = F(c_f)$. Moreover,*

$$cf(c) - F(c) \begin{cases} > 0, & 0 < c < c_f, \\ = 0, & c = c_f, \\ < 0, & c > c_f, \end{cases} \quad f'(c_f) < 0. \quad (30)$$

Proof. Let $D(c) = cf(c) - F(c)$, with $D(0) = 0$. For $c > 0$, $D'(c) = cf'(c)$. Single-peakedness makes D strictly increasing on $(0, c_m)$ and strictly decreasing on (c_m, ∞) , with $D(c_m) > 0$. For sufficiently large c , the density is decreasing on $[c/2, c]$, so

$$\frac{c}{2}f(c) \leq \int_{c/2}^c f(u) du \rightarrow 0.$$

Consequently $cf(c) \rightarrow 0$ and $D(c) \rightarrow -1$. Continuity and strict monotonicity on the decreasing part give a unique zero $c_f > c_m$ and the signs in (30). Since c_f is above the mode, $f'(c_f) < 0$. $\square$

Proof of Theorem 1 and its accompanying properties.

Since f is continuous at zero and $f(0) = 0$, $F(c)/c \rightarrow 0$ as $c \downarrow 0$. Hence $R(c) \rightarrow \infty$ there. Also $R(c) \geq c$, so $R(c) \rightarrow \infty$ as $c \rightarrow \infty$. Direct differentiation gives

$$R'(c) = \frac{F(c) - cf(c)}{F(c)^2}.$$

Lemma 1 implies that R strictly decreases up to c_f and strictly increases thereafter. Its unique minimizer is therefore the cutoff defined in (6). Changing variables $A = F(c)$ gives the equivalent expression

$$b_f = \min_{0 < A < 1} \frac{Q(A)}{A},$$

which we use in the potential and cost-distribution arguments below.

A positive equilibrium has $c = bA > 0$, $A = F(c)$, and $R(c) = b$. Conversely, every solution of $R(c) = b$ generates a positive equilibrium. The shape and endpoint limits of R give exactly

one positive root at b_f and two for $b > b_f$, ordered as stated. They give the absence of positive roots below b_f as well. Zero solves the equilibrium equation at every b .

For later arguments, note that the two positive cutoffs lie on opposite sides of c_f , where R' is nonzero. The implicit function theorem therefore makes both cutoff branches, and hence both adoption branches, continuously differentiable for $b > b_f$. Since R diverges at both endpoints and has the unique minimum b_f , both cutoffs converge to c_f as $b \downarrow b_f$. Thus both adoption shares converge to A_f . These continuity and differentiability properties justify the envelope comparison and the price limits used below; no formula for the branch derivatives is needed.

For the potential claims, differentiation gives (8). Since $Q(A)/A \rightarrow \infty$ at zero, the derivative $\mathcal{P}_A$ is negative in a right neighborhood of zero. Thus zero is a strict one-sided local maximum. At $b > b_f$, the signs of $\mathcal{P}_A = bA - Q(A)$ on the intervals separated by A_- and A_+ are respectively negative, positive, and negative. The lower root is a strict local minimum, the upper root a strict local maximum, and these are the only positive stationary points.

If $b \leq b_f$, $Q(A) \geq b_f A \geq bA$, with equality possible only at $(A, b) = (A_f, b_f)$. Hence $\mathcal{P}$ is strictly decreasing on $[0, 1]$, despite its zero derivative at A_f when $b = b_f$. The fold point is neither a local minimum nor a local maximum. Since $\mathcal{P}(0; b) = 0$, zero is the unique global maximizer in this case.

Finally, for $A > F(b)$ one has $Q(A) > b \geq bA$, so the potential is strictly decreasing beyond $F(b)$. Its global maximizers therefore lie in $[0, F(b)]$, where it is finite and continuous. This proves existence and compactness of $\mathcal{M}(b)$, including when $\mathcal{C}(1) = \infty$. For $b > b_f$, the derivative signs already established show that only zero and $A_+(b)$ can be global maximizers. $\square$

A.2 Noiseless Basins and Launch

Proof of Proposition 1. Let $H(A; b) = F(bA) - A$. On $[0, 1]$ it is continuously differentiable, $H(0; b) = 0$, and $H(1; b) = F(b) - 1 < 0$, so the interval is forward invariant and the differential equation has a unique solution. At zero, $H_A(0; b) = bf(0) - 1 = -1$. For $0 < A < 1$, strict monotonicity of F gives

$$\text{sgn } H(A; b) = \text{sgn}(bA - Q(A)).$$

The root count in Theorem 1 gives all basins by the one-dimensional phase line. Below the fold, $H < 0$ for all $A > 0$. Above it, the signs on the three intervals separated by A_- and A_+ are negative, positive, and negative. Solutions are monotone toward the stated limits, and uniqueness prevents crossing a stationary point. At the fold, $H < 0$ on both sides of A_f . Initial shares below A_f therefore converge to zero, while those above it decrease toward A_f ; an initial share exactly equal to A_f stays there. $\square$

Proof of Corollary 1. Fix $A_0 \in (0, A_f)$. On this interval, $Q(A)/A = R(Q(A))$ decreases strictly from infinity to b_f . The inequality $b > Q(A_0)/A_0$ is equivalent to $F(bA_0) > A_0$. It holds exactly when $A_-(b) < A_0 < A_+(b)$; here $A_0 < A_f$ ensures that the upper inequality is automatic once the positive roots exist. Proposition 1 gives convergence to the high root. At equality, A_0 is the lower stationary point. For $b_f < b < b_{\text{initial share}}(A_0)$, the initial share lies below $A_-(b)$ and converges to zero. For $b \leq b_f$, convergence to zero follows from $A_0 < A_f$ and Proposition 1. This proves necessity and sufficiency. The shape and endpoint limit of $Q(A)/A$ establish the comparative statics of the threshold in (13). $\square$

A.3 The Invariant Distribution and Its Concentration

Proof of Theorem 2 and the expected-potential limit. Step 1: exact potential and invariant probabilities. If a_i changes from zero to one, with others fixed, the change in Φ_n is $(b/n) \sum_{j \neq i} a_j - c_{i,n} = g_{i,n}(\mathbf{a}_{-i})$. For adjacent profiles $\mathbf{a}, \mathbf{a}'$ differing by that change, let $q_{n,\eta}(\mathbf{a}, \mathbf{a}')$ denote the continuous-time transition rate. Unit-rate individual revision gives

$$\frac{q_{n,\eta}(\mathbf{a}, \mathbf{a}')}{q_{n,\eta}(\mathbf{a}', \mathbf{a})} = \frac{L(g_{i,n}(\mathbf{a}_{-i}); \eta)}{1 - L(g_{i,n}(\mathbf{a}_{-i}); \eta)} = \exp \left\{ \frac{\Phi_n(\mathbf{a}'; b) - \Phi_n(\mathbf{a}; b)}{\eta} \right\}.$$

Thus (17) satisfies detailed balance. Positive rates for all single-coordinate changes establish irreducibility, so the finite continuous-time chain has this unique invariant distribution and converges to it from every initial profile.

The chain observed at revision opportunities chooses one institution uniformly, including opportunities that leave its action unchanged. If its transition kernel is $K_{n,\eta}$, the continuous-time generator is $n(K_{n,\eta} - I)$. Hence the two processes have identical invariant probabilities, as stated in Section 4.2.

Step 2: approximation of the best potential at each share. At a fixed number k of adopters, network benefits depend only on k , so the potential is highest when the k lowest costs adopt. Its normalized value is

$$\widehat{\mathcal{P}}_n(k) = \frac{b}{2} \frac{k}{n} \left(\frac{k}{n} - \frac{1}{n} \right) - C_n(k), \quad C_n(k) = \frac{1}{n} \sum_{i=1}^k Q\left(\frac{i}{n+1}\right). \quad (31)$$

Let $Q_n(u) = Q(\lceil nu \rceil / (n+1))$ for $u \in (0, 1]$ and $Q_n(0) = 0$. Then $C_n(k) = \int_0^{k/n} Q_n(u) du$, and $Q_n \rightarrow Q$ uniformly on compact subintervals of $[0, 1)$. For any sequence $k_n/n \rightarrow A < 1$, uniform convergence and boundedness on a slightly larger compact interval imply $C_n(k_n) \rightarrow \mathcal{C}(A)$ and

$\widehat{\mathcal{P}}_n(k_n) \rightarrow \mathcal{P}(A; b)$. For a sequence with limit $A = 1$, nonnegativity and Fatou's lemma give

$$\liminf_n C_n(k_n) = \liminf_n \int_0^1 \mathbf{1}\{u \leq k_n/n\} Q_n(u) du \geq C(1).$$

Consequently, for every limiting share $A \in [0, 1]$,

$$\limsup_n \widehat{\mathcal{P}}_n(k_n) \leq \mathcal{P}(A; b) \quad \text{whenever } k_n/n \rightarrow A, \quad (32)$$

with equality for $A < 1$. This covers infinite mean costs as well.

Step 3: a potential gap and concentration. The extended-valued potential is upper semicontinuous on $[0, 1]$ and has finite maximum $G = \max_A \mathcal{P}(A; b)$. As established after Theorem 1, its maximizing set is nonempty, compact, and contained in $[0, F(b)]$. Approximating a maximizer by k_n/n gives $\liminf_n \max_k \widehat{\mathcal{P}}_n(k) \geq G$. Compactness and (32) give the reverse inequality, so $\max_k \widehat{\mathcal{P}}_n(k) \rightarrow G$.

Fix $\delta > 0$ and let $D_\delta = \{A \in [0, 1] : \text{dist}(A, \mathcal{M}(b)) \geq \delta\}$. If this set is empty, concentration is immediate. Otherwise, upper semicontinuity on the compact set gives a strict gap: for some $g > 0$, $\mathcal{P}(A; b) \leq G - g$ on D_δ . If all the values there are $-\infty$, any finite $g > 0$ suffices. A subsequence argument using (32) shows that, for all sufficiently large n ,

$$\max_{k: k/n \in D_\delta} \widehat{\mathcal{P}}_n(k) \leq G - g/2, \quad \max_k \widehat{\mathcal{P}}_n(k) \geq G - g/4,$$

where an empty constrained maximum can be omitted. Every profile has potential no greater than the lowest-cost profile with the same share. Thus the unnormalized potential gap between any profile with adoption in D_δ and a global maximizing profile is at least $ng/4$. There are at most 2^n profiles. Bounding the Gibbs numerator by that count and the denominator by a single maximizing profile yields

$$\mu_{n, \eta_n}(A_n(\mathbf{a}) \in D_\delta) \leq \exp \left\{ n \log 2 - \frac{ng}{4\eta_n} \right\} \rightarrow 0.$$

This proves (18) without using the later characterization of $\mathcal{M}(b)$.

Step 4: expected potential. Let $Z_{n, \eta}$ be the denominator in (17), and let

$$S_{n, \eta} = - \sum_{\mathbf{a} \in \{0, 1\}^n} \mu_{n, \eta}(\mathbf{a}) \log \mu_{n, \eta}(\mathbf{a})$$

be the entropy of the invariant distribution. The Gibbs formula gives $\log Z_{n, \eta} = \mathbb{E}_\mu[\Phi_n]/\eta + S_{n, \eta}$.

Since $\log Z_{n,\eta} \geq \max_{\mathbf{a}} \Phi_n(\mathbf{a}; b)/\eta$ and $S_{n,\eta} \leq n \log 2$,

$$0 \leq \max_k \widehat{\mathcal{P}}_n(k) - \mathbb{E}_{\mu_{n,\eta}}[\Phi_n(\mathbf{a}; b)/n] \leq \eta \log 2.$$

Together with $\max_k \widehat{\mathcal{P}}_n(k) \rightarrow G$, this proves the expected-potential limit stated after the theorem. $\square$

A.4 Finite-Horizon Consistency of the Population Descriptions

The renewal laws in Section 4.1 are defined directly from continuum revision flows. The following result verifies that they also approximate the finite system in Section 4.2 on every fixed time horizon. It supplies the connection used in the order-of-limits discussion, rather than a prerequisite for the continuum formulation or the stationary selection theorem. Initial profiles may have any cost composition; only their aggregate shares are required to converge.

For this subsection, let $\mathbf{a}_{n,\eta}(t)$ denote the action-profile process generated by the revision protocol in Section 4.2, and write

$$A_{n,\eta}(t) = A_n(\mathbf{a}_{n,\eta}(t))$$

for its aggregate adoption share at time t .

Lemma 2 (Finite-horizon renewal limit). *Maintain Assumption 1 and the finite-population protocol in Section 4.2. Fix $b > 0$, $A_0 \in [0, 1]$, and a finite horizon $T > 0$. For fixed $\eta > 0$, suppose $A_{n,\eta}(0) \xrightarrow{p} A_0$. Then*

$$\sup_{0 \leq t \leq T} |A_{n,\eta}(t) - A(t; \eta)| \xrightarrow{p} 0,$$

where $A(\cdot; \eta)$ solves (11) with $A(0; \eta) = A_0$. For every sequence $\eta_n > 0$ with $\eta_n \rightarrow 0$, if $A_{n,\eta_n}(0) \xrightarrow{p} A_0$, then

$$\sup_{0 \leq t \leq T} |A_{n,\eta_n}(t) - A(t)| \xrightarrow{p} 0,$$

where A solves (12) with initial share A_0 . Also, $A(\cdot; \eta) \rightarrow A(\cdot)$ uniformly on $[0, T]$ as $\eta \downarrow 0$ when both paths start at A_0 .

Proof. Let $\bar{f} = f(c_m) < \infty$, the upper bound on the density, and extend F to be zero on negative arguments. Let F_n be the empirical CDF of the quantile costs in (14). Their construction implies

$$\varepsilon_n \equiv \sup_{c \in \mathbb{R}} |F_n(c) - F(c)| \leq \frac{1}{n}.$$

For the proof, define $S_\eta(A) = \int L(bA - c; \eta) dF(c)$ for real A and $H_\eta(A; b) = S_\eta(A) - A$. If Z has CDF $L(y; 1) = (1 + e^{-y})^{-1}$, then

$$S_\eta(A) = \mathbb{E}[F(bA - \eta Z)].$$

The extended CDF is globally $\bar{f}$ -Lipschitz. Thus S_η is $b\bar{f}$ -Lipschitz, uniformly in η , and

$$\sup_{A \in [0,1]} |S_\eta(A) - F(bA)| \leq \bar{f}\eta \mathbb{E}|Z| = 2\bar{f}\eta \log 2. \quad (33)$$

Here $\mathbb{E}|Z| = 2 \int_0^\infty (1 + e^y)^{-1} dy = 2 \log 2$. Consequently both H_η and $H_0(A; b) = F(bA) - A$ are $(1 + b\bar{f})$ -Lipschitz. Their vector fields point into $[0, 1]$ at its endpoints, so their solutions exist uniquely and remain in that interval.

At profile $\mathbf{a}$, the exact conditional drift of aggregate adoption is

$$d_{n,\eta}(\mathbf{a}) = \frac{1}{n} \sum_{i=1}^n \left[L\left(b \left(A_n(\mathbf{a}) - \frac{a_i}{n} \right) - c_{i,n}; \eta \right) - a_i \right].$$

For any real A , integration by parts against the bounded monotone function $c \mapsto L(bA - c; \eta)$ gives

$$\left| \int L(bA - c; \eta) dF_n(c) - S_\eta(A) \right| \leq \varepsilon_n.$$

For $A = A_n(\mathbf{a})$, the average of the logit terms in the drift is bounded above by the empirical expression at A and below by that expression at $A - 1/n$. The uniform Lipschitz bound for S_η then implies

$$\sup_{\mathbf{a}} |d_{n,\eta}(\mathbf{a}) - H_\eta(A_n(\mathbf{a}); b)| \leq \varepsilon_n + \frac{b\bar{f}}{n}. \quad (34)$$

Combining (33) and (34) gives a bound relative to the noiseless drift that is uniform in the profile:

$$\sup_{\mathbf{a}} |d_{n,\eta}(\mathbf{a}) - H_0(A_n(\mathbf{a}); b)| \leq \varepsilon_n + \frac{b\bar{f}}{n} + 2\bar{f}\eta \log 2.$$

The aggregate finite process has the decomposition

$$A_{n,\eta}(t) = A_{n,\eta}(0) + \int_0^t d_{n,\eta}(\mathbf{a}_{n,\eta}(s)) ds + M_{n,\eta}(t),$$

where $M_{n,\eta}$ is a martingale starting at zero. An aggregate jump has magnitude at most $1/n$, and total revision opportunities arrive at rate n . Its expected quadratic variation up to T is therefore

at most T/n , and the martingale maximal inequality gives

$$\mathbb{E} \left[\sup_{0 \leq t \leq T} |M_{n,\eta}(t)|^2 \right] \leq \frac{4T}{n}.$$

Comparing the decomposition with the integral equation for $A(\cdot; \eta)$ and applying Gronwall's inequality yields

$$\begin{aligned} \sup_{t \leq T} |A_{n,\eta}(t) - A(t; \eta)| &\leq e^{(1+b\bar{f})T} \left(|A_{n,\eta}(0) - A_0| + T \left(\varepsilon_n + \frac{b\bar{f}}{n} \right) \right. \\ &\quad \left. + \sup_{t \leq T} |M_{n,\eta}(t)| \right). \end{aligned}$$

This proves the fixed-noise approximation. Using the noiseless drift comparison instead adds $2T\bar{f}\eta_n \log 2$ to the bound and proves the joint large-population, vanishing-noise result for any such sequence. Finally, the same deterministic comparison gives

$$\sup_{t \leq T} |A(t; \eta) - A(t)| \leq 2T\bar{f}\eta \log 2 e^{(1+b\bar{f})T},$$

proving the last claim. All statements concern fixed T ; none exchanges this limit with an unbounded time horizon. $\square$

A.5 The Equilibrium-Selection Threshold

Proof of Theorem 3 and (21). Section 3 established that zero is the unique potential maximizer when $b \leq b_f$. For $b > b_f$, the only candidates are zero and $A_+(b)$. Let $V(b) = \mathcal{P}(A_+(b); b)$ and extend it continuously to b_f by $V(b_f) = \mathcal{P}(A_f; b_f)$. Since $Q(u) > b_f u$ on $(0, A_f)$,

$$V(b_f) = \frac{b_f A_f^2}{2} - \int_0^{A_f} Q(u) du < 0.$$

For $b > b_f$, the first-order condition at $A_+(b)$ gives the envelope derivative

$$V'(b) = \frac{A_+(b)^2}{2} \geq \frac{A_f^2}{2} > 0.$$

Thus $V(b) \rightarrow \infty$ and has a unique zero $b_p > b_f$. Its sign gives (19). The positive maximizer at equality is $A_p = A_+(b_p) > A_f$.

At b_p , the potential is nonpositive for every share, with equality at zero and the unique

positive maximizer A_p . For every $A > 0$,

$$\frac{2\mathcal{C}(A)}{A^2} \geq b_p,$$

with equality exactly at A_p . This proves (20) and uniqueness of its minimizer. Finally, strict monotonicity of Q implies

$$\mathcal{C}(A_f) < A_f Q(A_f) = b_f A_f^2.$$

Evaluating the minimum in (20) at A_f therefore gives $b_p \leq 2\mathcal{C}(A_f)/A_f^2 < 2b_f$. This proves (21). The stationary-adoption statements follow by applying Theorem 2 to the characterized maximizing set. $\square$

A.6 Debt Corridors and Comparative Statics

Proof of Corollary 2.

Under Assumption 2, since $b_p > b_f$, $\mathcal{X}_p \subseteq \mathcal{X}_f$. If $b_{\max} > b_p$, raising the threshold from b_f to b_p moves both endpoints strictly inward, so the inclusion is strict. The case $b_f < b_{\max} < b_p$ follows directly from the level-set description.

Corollary 1 gives the launch requirement; strict single-peakedness gives its open-interval form whenever peak strength strictly exceeds the initial-share threshold.

If $\tilde{F} \geq F$, then $\tilde{Q} \leq Q$ and $\tilde{\mathcal{C}} \leq \mathcal{C}$. Taking minima in (6) and (20) gives weakly lower existence and selection thresholds, whose superlevel sets therefore expand. Scaling costs by κ scales Q and $\mathcal{C}$ by κ , without changing the minimizing shares in those two formulas. Hence b_f, b_p scale by κ , while A_f, A_p remain unchanged. $\square$

A.7 Network Surplus and Corrective Transfers

The surplus identities in Section 5.2. For a fixed share A , total gross network benefit is bA^2 regardless of which institutions adopt. Lowest-cost assignment minimizes its implementation cost, giving (23). The identity $U(A; b) = \mathcal{P}(A; 2b)$ and Theorem 3 imply (24) and the complete maximizing-set description. The positive-surplus condition also follows directly from $U(A; b) = A^2[b - \mathcal{C}(A)/A^2]$ for $A > 0$. At $b = b_w$, zero and A_p attain the maximum value zero; above this threshold, $A_+(2b)$ uniquely maximizes surplus at a strictly positive value. Since $b_p < 2b_f$, $b_w < b_f$. Finally, differentiating (23) and subtracting (8) gives the externality bA at the marginal adopter. $\square$

Proof of Proposition 2 and the launch comparison. Outside taxes do not depend on an institution's action. Adding (25) therefore replaces b by $(1 + \lambda)b$ in the private equilibrium equation and in

the finite-player gain. Theorems 1, 2, and 3 give the existence and selection conditions with the original costs.

Transfers and outside taxes cancel in aggregate. At $\lambda = 1$, (26) equals $U(A; b)$ for every share. The same identity holds at the finite-profile level: the full-correction potential is

$$\Phi_n(\mathbf{a}; 2b) = \frac{2b}{n} \sum_{i < j} a_i a_j - \sum_i c_{i,n} a_i = \sum_i a_i \left(\frac{b}{n} \sum_{j \neq i} a_j - c_{i,n} \right),$$

which is the sum of the unsubsidized institutional payoffs, and hence net surplus after cancelling the policy transfers. Applying the concentration and expected-potential conclusions of Theorem 2 at strength $2b$ shows that stationary aggregate adoption concentrates on $\arg \max_A U(A; b)$ and stationary expected surplus per institution tends to its maximum. In particular, this conclusion accounts for the cost composition of adoption, not only its aggregate share. The maximizing sets at, above, and below b_w follow from (24); no mixture weights at equality are asserted.

For the launch comparison, holding the transfer in place replaces b by $(1 + \lambda)b$ in the noiseless renewal law as well. For $A_0 \in (0, A_f)$, Corollary 1 therefore gives convergence to the subsidized high equilibrium exactly when

$$(1 + \lambda)b > \frac{Q(A_0)}{A_0},$$

which is (27). At equality, A_0 is the subsidized lower equilibrium rather than a point in the high equilibrium's basin. For a fixed $b > b_f$, the subsidized critical initial share is $A_-((1 + \lambda)b)$. The function $Q(A)/A = R(Q(A))$ is strictly decreasing on $(0, A_f)$, so its lower-branch solution falls as λ rises. This proves the basin expansion. At $A_0 = 0$, the renewal drift remains zero for every λ because $F(0) = 0$. $\square$

A.8 Convenience Value

Proof of Corollary 3. By Assumption 2 and $b_{\max} > b_f$, the feasibility corridor is a nondegenerate compact interval. Its upper endpoint satisfies $b(x_U(b_f); \varphi) = b_f$. Approaching this boundary from within the feasibility corridor gives $b(x) \downarrow b_f$ and $A_+(b(x)) \rightarrow A_f$, by the continuity established in the proof of Theorem 1. Immediately to its right, $b(x) < b_f$, so zero is the only adoption equilibrium. Payoff retention $s(x; \varphi)$ is continuous at $x_U(b_f)$. Subtracting the two one-sided limits of (28) therefore cancels the retained cash payoff and leaves $\beta h A_f$, as in (29). $\square$

B Allowing Positive Density at Zero

This section relaxes the requirement that the cost density vanish at zero. We retain all other conditions in Assumption 1, including the strictly interior mode $c_m > 0$ and the strict increase

and decrease of the density on either side of that mode. We focus on the additional case $0 < f(0) < \infty$, since $f(0) = 0$ is covered by the main analysis. The condition $F(0) = 0$ is maintained: a positive density at zero does not introduce a positive mass of costless institutions.

Define

$$b_0 \equiv \frac{1}{f(0)}. \quad (35)$$

This additional threshold determines whether a sufficiently small positive network expands under noiseless renewal. The existence threshold and the stationary-selection threshold remain distinct, and both lie strictly below b_0 .

B.1 Equilibrium Existence and Stationary Selection

The definitions of c_f , b_f , and A_f in (6) remain well defined. The only change in the positive-equilibrium count is that the lower branch now reaches zero at a finite benchmark strength.

Proposition 3 (Positive density at zero). *Under the conditions of this section:*

- (i) *The function $c/F(c)$ has a unique interior minimizer c_f , with $c_f > c_m$ and $b_f < b_0$. Zero adoption is always an equilibrium. The positive equilibria are*

$$\{A \in (0, 1) : A = F(bA)\} = \begin{cases} \emptyset, & 0 < b < b_f, \\ \{A_f\}, & b = b_f, \\ \{A_-(b), A_+(b)\}, & b_f < b < b_0, \\ \{A_+(b)\}, & b \geq b_0. \end{cases}$$

The lower branch satisfies $A_-(b) \rightarrow 0$ as $b \uparrow b_0$. The upper branch is defined for every $b > b_f$ and satisfies $A_+(b) \rightarrow A_f$ as $b \downarrow b_f$.

- (ii) *The potential-maximizer characterization in (19) remains valid. Its threshold is*

$$b_p = \min_{0 < A < 1} \frac{2C(A)}{A^2},$$

with the minimum attained uniquely at $A_p = A_+(b_p)$. Moreover,

$$b_f < b_p < \min\{2b_f, b_0\}, \quad A_p > A_f. \quad (36)$$

Proof. Write $R(c) = c/F(c)$ and $D(c) = cf(c) - F(c)$. Continuity of f at zero gives

$$\lim_{c \downarrow 0} \frac{F(c)}{c} = f(0), \quad \lim_{c \downarrow 0} R(c) = b_0.$$

Also, $R(c) \rightarrow \infty$ as $c \rightarrow \infty$.

The single-crossing argument does not require $f(0) = 0$. Indeed, $D(0) = 0$ and

$$D'(c) = cf'(c).$$

Thus D increases strictly up to c_m and decreases strictly thereafter. Since the density is eventually decreasing and integrable, $cf(c) \rightarrow 0$, so $D(c) \rightarrow -1$. There is consequently a unique $c_f > c_m$ with $D(c_f) = 0$. Because

$$R'(c) = -\frac{D(c)}{F(c)^2},$$

R decreases strictly from its limiting value b_0 to b_f on $(0, c_f)$ and increases strictly from b_f to infinity on (c_f, ∞) . In particular, $b_f < b_0$.

Positive adoption equilibria correspond exactly to solutions of $R(c) = b$, with adoption share $F(c)$. The decreasing part of R takes precisely the values in (b_f, b_0) , whereas its increasing part takes all values above b_f . This proves the root count and the branch limits in part (i). The fold remains nondegenerate because $f'(c_f) < 0$.

For part (ii), write $Q = F^{-1}$. The potential derivative remains

$$\mathcal{P}_A(A; b) = bA - Q(A).$$

For $b \leq b_f$, it is negative except possibly at A_f when $b = b_f$, so zero is the unique global maximizer. For $b_f < b < b_0$, the potential decreases, increases, and then decreases on the intervals separated by $A_-(b)$ and $A_+(b)$. Its only global candidates are therefore zero and $A_+(b)$. For $b \geq b_0$, the potential increases on $(0, A_+(b))$ and decreases thereafter, so $A_+(b)$ is its unique global maximizer.

Define

$$V(b) = \mathcal{P}(A_+(b); b), \quad b > b_f,$$

and extend it continuously to b_f . Since $Q(u) > b_f u$ on $(0, A_f)$,

$$V(b_f) = \frac{b_f A_f^2}{2} - \int_0^{A_f} Q(u) du < 0.$$

Along the upper branch, the first-order condition gives

$$V'(b) = \frac{A_+(b)^2}{2} > 0.$$

To locate the zero of V relative to b_0 , use the strict increase of f near the origin. For $0 < c < c_m$,

$$F(c) = \int_0^c f(u) du > f(0)c.$$

Equivalently, $Q(A) < b_0 A$ for $0 < A < F(c_m)$. Hence, for every such A ,

$$\mathcal{P}(A; b_0) = \frac{b_0 A^2}{2} - \int_0^A Q(u) du > 0.$$

It follows that $V(b_0) > 0$. Continuity and strict monotonicity therefore give a unique $b_p \in (b_f, b_0)$ at which $V(b_p) = 0$. The sign of V establishes (19), and $A_p = A_+(b_p) > A_f$.

At b_p , the potential is nonpositive everywhere and equals zero only at 0 and A_p . Thus

$$\frac{2\mathcal{C}(A)}{A^2} \geq b_p \quad \text{for every } A \in (0, 1),$$

with equality only at A_p . This proves the stated characterization of b_p . Finally, since Q is strictly increasing,

$$\mathcal{C}(A_f) < A_f Q(A_f) = b_f A_f^2.$$

Evaluating the minimum defining b_p at A_f gives $b_p < 2b_f$, completing (36). $\square$

The distinction between existence and selection is therefore unchanged. A positive network first becomes feasible at b_f , but it wins the global potential comparison only at the higher threshold b_p . In particular, the whole interval (b_f, b_p) lies below b_0 , where zero and high adoption are both locally persistent. Positive density at zero changes what happens at sufficiently high quality, not the coexistence underlying this gap.

B.2 Noiseless Adjustment and Launching

The renewal law remains

$$\dot{A} = F(bA) - A.$$

Its derivative at zero is now

$$\left. \frac{\partial}{\partial A} [F(bA) - A] \right|_{A=0} = bf(0) - 1.$$

Zero adoption is therefore locally asymptotically stable when $b < b_0$ and unstable when $b > b_0$. At $b = b_0$, the derivative is zero, but strict increase of the density near the origin gives

$$F(b_0 A) > f(0)b_0 A = A$$

for all sufficiently small $A > 0$. Thus zero is unstable from the right at $b = b_0$ as well. Correspondingly, zero is a strict one-sided local potential maximum for $b < b_0$ and a strict one-sided local minimum for $b \geq b_0$.

For $b < b_f$ and at $b = b_f$, the convergence conclusions in Proposition 1 are unchanged. For $b_f < b < b_0$, its two-basin description also remains unchanged: initial adoption below $A_-(b)$ converges to zero, while initial adoption above $A_-(b)$ converges to $A_+(b)$. For $b \geq b_0$, however, the lower positive equilibrium is absent and

$$A(0) > 0 \implies \lim_{t \rightarrow \infty} A(t) = A_+(b).$$

Indeed, the renewal drift is positive below the unique positive root and negative above it. An initial share exactly equal to zero still stays at zero because $F(0) = 0$. Instability means that a small positive initial share grows, not that the noiseless process leaves zero spontaneously.

The launch condition in Corollary 1 retains its form. For $A_0 \in (0, A_f)$, convergence to the high equilibrium occurs if and only if

$$b > \frac{F^{-1}(A_0)}{A_0}.$$

This threshold decreases strictly with A_0 , but its limit is now finite:

$$b_f < \frac{F^{-1}(A_0)}{A_0} < b_0, \quad \lim_{A_0 \downarrow 0} \frac{F^{-1}(A_0)}{A_0} = b_0. \quad (37)$$

These conclusions follow from the decreasing part of R and the phase-line argument above. At equality, the initial share is the lower equilibrium and does not converge to the high one.

Economically, a positive density at zero supplies enough very low-cost institutions for an arbitrarily small positive network to expand when quality is sufficiently high. The assumption $f(0) = 0$ excludes this possibility at every finite quality. It is useful for obtaining a positive critical mass throughout the high-quality region, but is not necessary for a critical mass near the existence and selection thresholds.

B.3 Implications for the Remaining Results

Stochastic selection and population approximation. Theorem 2 continues to hold under the relaxed assumptions. The exact-potential identity, detailed balance, and Gibbs invariant distribution do not use $f(0) = 0$. The large-population concentration proof uses the continuity of the inverse distribution, the approximation of cutoff costs, and a potential gap away from the global maximizing set. These properties remain valid. In particular, the potential still attains its

maximum in $[0, F(b)]$, and the proof accommodates infinite mean costs.

Lemma 2 is unchanged as well. Its drift and smoothing estimates use the bounded density $\sup_c f(c) = f(c_m)$, not a zero density at the origin. Thus the continuum renewal law and the finite-population stationary comparison retain their common revision foundation.

Debt corridors, surplus, and corrective transfers. The feasibility and coordination corridors remain determined by b_f and b_p , respectively. Their nesting and the cost-distribution comparative statics follow from the same threshold formulas. Under the maintained quality and pricing conditions, the branch-continuation price loss at the upper fold also remains $\beta h A_f$: the positive branch still ends at the strictly positive share A_f .

The identity $U(A; b) = \mathcal{P}(A; 2b)$ is unaffected, so the social threshold remains

$$b_w = \frac{b_p}{2} < b_f.$$

A matching transfer still replaces private strength b by $(1 + \lambda)b$, and full correction still aligns the potential with network surplus. Consequently, the existence and stationary-implementation conclusions of Proposition 2 remain valid.

Temporary support can still help launch a self-sustaining network in the coexistence region $b_f < b < b_0$. Starting from $A_0 \in (0, A_-(b))$, any admissible matching rate satisfying

$$(1 + \lambda)b > \frac{F^{-1}(A_0)}{A_0}$$

makes adoption grow toward the subsidized high equilibrium. That equilibrium lies above $A_f > A_-(b)$, so adoption crosses the unsubsidized critical mass after a finite time. Withdrawing support strictly after that crossing leads to the unsubsidized high equilibrium, as in the main analysis. For $b \geq b_0$, by contrast, every strictly positive initial share already converges to the high equilibrium without a transfer.

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