

Who Keeps a Bubble Alive?

Skin in the Game and the Survival of Rational Bubbles

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October 21, 2026

Abstract

We study how retained ownership supports the value of a zero-dividend asset held for resale. A sponsor chooses hidden maintenance that determines market survival, bears full cost, and captures benefits only through retained tokens. A positive stationary equilibrium exists if and only if retention exceeds an endogenous minimum, and extra curvature restrictions yield two equilibria. Conditional on survival, prices below the upper equilibrium converge to the lower one. Credible news that future retention falls below the minimum causes immediate collapse through backward unraveling. Thus, insider unlocks affect valuation via governance without changing token supply or cash flows.

Keywords: rational bubbles; lockups; tokenomics; skin in the game.

JEL Classification: G12, G32, D82, G34.

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1 Introduction

Who has an incentive to keep a market open when the asset traded in it promises no cash flows? If continued resale depends on infrastructure maintained by an issuer, foundation, or development team, the party that bears the cost of upkeep need not capture its full benefit. Other holders can sell into the market without paying for its preservation. Retained ownership can therefore determine whether a positive resale value can be sustained at all.

Crypto and platform tokens make this friction observable. They often pay no dividend, so their price is a price of resale, and resale requires a venue for listing, matching, and, in many cases, delegated market making. Those activities are costly and are typically controlled by a foundation or core team that also holds a locked allocation. [Howell et al. \(2020\)](#) study more than 1,500 initial coin offerings. In their sample, 22 percent of issuers impose a vesting schedule on insider tokens and 43 percent set tokens aside for a foundation, bounty, or endowment. They interpret insider vesting, following [Brav and Gompers \(2003\)](#) on IPO lock-ups, as a commitment device. Some contracts make the lock-up part of the venue itself. Bancor reserved funds for a market maker charged with maintaining price stability, funds that could not be removed for a pre-specified period ([Howell et al., 2020](#)).

Throughout, we study a pure resale asset whose value depends on continued access to a functioning secondary market. We call the asset a *token*, while abstracting from the transaction services, redemption rights, and profit claims that some actual tokens provide. The same mechanism applies to any store of value whose payoff is the continuation of a resale institution.

The theory of zero-dividend assets takes that continuation as given. In [Tirole \(1985\)](#) resale succeeds with certainty. In [Weil \(1987\)](#), building on [Samuelson \(1958\)](#), the market remains open each period with an exogenous probability that Weil calls confidence. Because nobody chooses whether the venue remains open, these models cannot explain when confidence is incentive-compatible, nor distinguish a market whose support is locked in

from one whose support is about to be released. The corporate-finance analysis of ownership and lock-ups explains how retention disciplines a noncontractible action that affects cash flows or listing quality (Jensen and Meckling, 1976; Chemla and Hennessy, 2014; Brav and Gompers, 2003). For a pure resale asset the value of the incentive instrument is itself endogenous to the action it is meant to motivate. The sponsor maintains the market because its retained tokens are valuable, and the tokens are valuable because investors expect the sponsor to maintain the market.

We ask whether this feedback can support an asset that offers only the prospect of resale, and what happens when the retained exposure is released. The mechanism operates without a change in total token supply, leverage, or fundamental cash flows. Instead, it turns on the exposure retained by those responsible for maintaining the resale market. IPO lock-ups offer a useful comparison, as their expiration allows insiders to sell. Field and Hanka (2001) examine 1,948 such agreements and find a three-day abnormal return of -1.5 percent. Unlike a pure resale asset, however, equity also represents a claim on the firm's cash flows.

Token-unlock episodes illustrate how prices can respond before locked holdings become transferable. In January 2023 the dYdX Foundation announced that transfer restrictions on tokens issued to investors, employees, and consultants would be extended, postponing the initial release from 3 February 2023 to 1 December 2023.¹ The token price rose when the delay became public. In late November 2023, DYDX, OP, and SUI fell ahead of large scheduled unlocks. These episodes motivate a question beyond the price impact of anticipated sales. If an unlock leads those responsible for maintaining the resale market to reduce their exposure, it may also weaken their incentive to keep that market operating. For a pure resale asset, the risk is the loss of the institution that makes resale possible, not only a lower price.

We characterize when retained ownership supports a positive price, how that restric-

¹dYdX Foundation, "Lock-up extension for \$DYDX issued to investors, employees and consultants," 24 January 2023, updated 31 January 2023, <https://www.dydx.foundation/blog/lock-up-extension>.

tion shapes live-path dynamics, and how an announced future release unravels current demand.

Our framework is an overlapping-generations exchange economy. At each active date, the young purchase a fixed supply of tokens from the old. A predetermined fraction of the young belongs to a sponsor partnership. Its members trade individually, but their holdings are subsequently locked and managed collectively for the maintenance decision. After trade, the sponsor chooses hidden maintenance that determines the probability of another round of resale. The sponsor bears the entire maintenance cost and internalizes only the benefit accruing to its retained share. Investors rationally anticipate this choice when buying the token. Survival is thus an incentive-compatible outcome, not an exogenous confidence parameter in [Weil \(1987\)](#).

The overlapping-generations structure isolates a recurring governance problem in which maintenance is controlled by a party whose exposure remains locked until the survival uncertainty is resolved. It does not require a new trading platform to be created each period. A continuing foundation or core team motivates the institutional interpretation, although the model does not describe a long-lived organization's full financing or contracting problem. Relative to [Tirole \(1985\)](#) and [Weil \(1987\)](#), the central change is that someone must have an incentive to preserve the resale opportunity.

The first result establishes a minimum-retention threshold. A positive stationary equilibrium exists if and only if retention exceeds this threshold. The threshold compares the marginal cost of maintenance with the continuation value consistent with household demand. Weil's minimum-confidence restriction remains a necessary demand-side condition, but confidence must now also be incentive compatible for the sponsor.

The second result concerns multiplicity and comparative statics. Under some convexity conditions, retention strictly above the threshold supports exactly two positive stationary equilibria. The lower equilibrium has a lower price and less maintenance, whereas the upper one has a higher price and more maintenance.

The third result characterizes equilibrium price dynamics conditional on survival. Every positive initial price below the upper stationary price converges to the lower stationary price. The upper stationary price is itself sustainable. A higher initial price would eventually require a resale payment that no future cohort can finance. The upper equilibrium therefore determines the largest feasible initial price. Greater retention expands the entire feasible price interval.

The fourth result concerns unlock announcements. Suppose investors learn that, from a known finite date onward, the retained share will be below the minimum-retention threshold. No positive price can then be supported after that date, whether stationary or not. The loss of future resale unravels backward, and the bubble collapses when the news becomes common knowledge. Smaller releases of retained exposure do not necessarily eliminate all positive prices. They can instead reduce the set of sustainable valuations.

Our contribution is to connect a corporate-finance incentive constraint to the existence and dynamics of a pure bubble. Token retention is not new as a governance device. What is distinctive here is that the sponsor protects no productive cash flow. Its stake is valuable only if another cohort is willing to buy. An unlock can therefore matter through the exposure of the party responsible for continued trading, even if total token supply is unchanged. Appendix B isolates the resulting wedge between the sponsor's incentives and the joint interests of current holders.

Figure 1 summarizes the feedback. Retained ownership gives the sponsor a reason to maintain the venue. Maintenance then supports survival, and survival supports the resale price. The resulting price determines the value of the sponsor's exposure, which is what makes maintenance privately worth undertaking. Passive holders benefit from maintenance without bearing its cost.

Section 2 relates the paper to the literature. Section 3 presents the model. Section 4 characterizes pricing, maintenance, stationary equilibria, and dynamics, then discusses the token episodes. Sections 5 and 6 discuss the interpretation and conclude. All proofs are

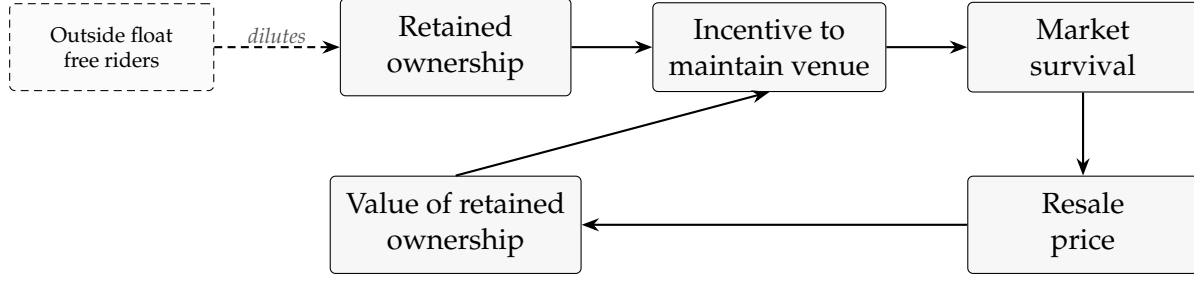


Figure 1: The maintenance and resale value.

Note: The retained share is fixed. Solid arrows show how its value feeds back into maintenance through survival and resale prices. The dashed arrow is free-rider dilution from unlocked holdings. A larger outside float reduces the share of the maintenance benefit internalized by the sponsor.

collected in Appendix D. Appendix A provides an effort foundation, Appendix B studies the governance externality, and Appendix C allows nonlinear old-age utility.

2 Related Literature

The paper connects ownership incentives and lockups in corporate finance to the valuation of pure resale assets. The separation between control and ownership creates an agency problem in Jensen and Meckling (1976). Limited managerial ownership is one of the agency frictions estimated by Nikolov and Whited (2014) and is related to cash policy. Retention can restore incentives when the relevant action is hidden, as in securitization (Chemla and Hennessy, 2014). Ownership structure also determines who monitors (Maug, 1998; Pagano and Röell, 1998). Moreover, Panousi and Papanikolaou (2012) show that insider ownership changes managers' private investment choices when those managers cannot fully diversify. Eisfeldt and Papanikolaou (2013) emphasize that financiers cannot fully own organization capital because key talent shares its cash flows. What cannot be fully appropriated here is the resale price, not the cash flows of organization capital. He and Krishnamurthy (2013) show that an intermediary's scarce capital can determine risk premia. Hu and Varas (2025) study intermediary financing under limited commitment. In those papers the constrained party's exposure affects prices through cash flows or

risk-bearing capacity. Here the sponsor's claim is only on resale, and the hidden margin is upkeep. Outside holders free-ride on hidden upkeep, and maintenance preserves only an opportunity to resell.

The lockup literature provides a particularly useful comparison. [Brav and Gompers \(2003\)](#) find support for a commitment explanation of IPO lockups. [Field and Hanka \(2001\)](#) document trading-volume increases and negative abnormal returns around lockup expirations. [Ofek and Richardson \(2003\)](#) connect the internet-stock boom and bust to heterogeneous beliefs, short-sale restrictions, and the release of insider shares. [Hong et al. \(2006\)](#) model how insider lockups and anticipated changes in float affect speculative resale values under disagreement and short-sale constraints. Here the relevant margin is the controller's incentive to preserve trading, not the speculative resale option generated by heterogeneous beliefs. This does not establish that illiquidity is always desirable, because informed blockholder trading can itself improve governance ([Edmans, 2009](#)). Rather than solve for an optimal lockup, we take the retention arrangement as given.

The second connection relates to token valuation and platform finance. [Cong et al. \(2021b, 2022\)](#) study token adoption, valuation, and platform financing. [Sockin and Xiong \(2023b,a\)](#) examine cryptocurrency demand and decentralization through tokenization. [Garratt and Wallace \(2018\)](#) emphasize self-fulfilling valuations of privately issued outside monies, and [Biais et al. \(2023a\)](#) derive cryptocurrency prices from net transactional benefits. Our benchmark removes those benefits to isolate the incentive requirements for continued resale.

Retained tokens already play an important role in financing and governance. [Gan et al. \(2021, 2023\)](#) analyze ICO design and incentives for platform building. [Chod et al. \(2022\)](#) study financing, governance, and moral hazard under tokenization. [Han et al. \(2026\)](#) examine strategic token trading and governance conflicts, including illiquidity and long-term incentives, and [Canidio \(2026\)](#) connects developers' retained tokens to investment incentives. These papers already show that retained tokens can discipline platform or

developer actions. We study a different object. The token has no service or cash-flow payoff, and the results are a retention floor, a feasible price interval, and unraveling after an unlock announcement for an asset whose value comes entirely from resale.

Related platform-design questions include digital tokens and platform building, corporate capture, tokens versus equity, staking, and token distribution ([Li and Mann, 2025](#); [Ferreira et al., 2023](#); [Malinova and Park, 2023](#); [Cong et al., 2025](#); [Canidio, 2023](#)). Empirically, [Howell et al. \(2020\)](#) study ICO financing and subsequent real outcomes, highlighting the importance of disclosure and credible project commitment. We do not model issuance proceeds or the choice between token and equity financing. Retention is an exogenous constraint on the sponsor’s exposure during the maintenance decision.

The feedback between asset value and infrastructure incentives also connects our analysis to blockchain security. [Pagnotta \(2022\)](#) jointly determines Bitcoin prices and network security. [Budish \(2025\)](#) studies the economic cost of supporting blockchain trust and the incentives against attack. [Biais et al. \(2019\)](#) and [Saleh \(2021\)](#) analyze consensus, including the role of miners’ and validators’ ownership. These papers already link an asset’s value to the incentives that keep its infrastructure running. In those models the threat is an attack or a fork, and the token often carries transaction utility or a protocol-funded reward. Neither is present here. The sponsor provides insufficient hidden upkeep. We abstract from the contracting and mining institutions studied by [Cong and He \(2019\)](#) and [Cong et al. \(2021a\)](#). [Makarov and Schoar \(2020\)](#) document persistent cross-exchange arbitrage in cryptocurrency markets, and evidence on wash trading cautions against using reported volume alone to measure activity ([Cong et al., 2023](#)). Those facts concern market segmentation and recorded volume. They are not the hidden-maintenance margin isolated here. [Biais et al. \(2023b\)](#) provide a broader perspective on this research area.

Finally, the model builds on rational-bubble theory. The starting point is the valuation of zero-dividend assets in overlapping generations ([Samuelson, 1958](#); [Tirole, 1985](#)). [Weil \(1987\)](#) introduces stochastic collapse and a minimum survival probability for stationary

existence. Santos and Woodford (1997) provide a general asset-pricing treatment, and Martín and Ventura (2018) review the broader theory. Kocherlakota (1992, 2008) and Hugonnier (2012) show that debt and portfolio constraints affect whether a bubble can exist. Subsequent work studies how bubbles interact with liquidity and investment (Farhi and Tirole, 2012), growth (Martín and Ventura, 2012), global imbalances (Ikeda and Phan, 2019), and monetary policy (Asriyan et al., 2021). Enders and Hakenes (2021) emphasize the wealth available to absorb a bubble. Household resources also bound resale payments here, but an additional constraint comes from the incentives to maintain the market.

Other theories connect bubbles and crises to leverage and risk shifting (Allen and Gale, 2000; Barlevy, 2014; Ikeda and Phan, 2016; Bengui and Phan, 2018; Allen et al., 2022), synchronization (Abreu and Brunnermeier, 2003), or the dynamics of asset quality (Fukui, 2018; Caramp, 2025). Goldstein et al. (2025) review related sources of financial fragility. Our channel requires neither debt nor a deterioration in the token’s fundamental payoff. It is the incentive-compatible probability of continued resale that changes. The lockup commits exposure, not the hidden action or future prices.

3 The Model

3.1 Demographics and Preferences

Time is discrete and runs forever. At each date t , a unit mass of agents is born and lives for two periods. Each generation is endowed with $e_1 > 0$ units of perishable consumption good when young and $e_2 \geq 0$ units when old. Preferences are

$$u(c_{it}^y) + \gamma E_t[c_{i,t+1}^o] - \text{maintenance disutility, if any,}$$

where any maintenance disutility is borne by the sponsor partnership introduced below.

Assumption 1a (Young-age utility). The function $u : (0, e_1] \rightarrow \mathbb{R}$ is twice continuously

differentiable and satisfies

$$u'(\cdot) > 0, \quad u''(\cdot) < 0, \quad \lim_{c \downarrow 0} u'(c) = +\infty.$$

Moreover,

$$u'(e_1) < \gamma. \tag{1}$$

Condition (1) is the baseline dynamic-inefficiency restriction. At the endowment allocation, the marginal utility cost of transferring one unit out of young-age consumption is smaller than its old-age utility benefit. It therefore leaves room for a risk-free store of value. The condition plays the role of the interest-rate-versus-growth-rate restriction in classical overlapping-generations bubble models (Samuelson, 1958; Tirole, 1985).

Old-age utility is linear, and $\gamma > 0$ measures the weight on old-age consumption. In Appendix C, we allow the old-age utility to be an arbitrary increasing concave function. All our results can go through under some additional technical assumptions.

3.2 The Token

There is a fixed unit supply of a divisible token. The token pays no dividend, provides no transaction service or collateral value, carries no redemption right, and conveys no claim on profits. Any positive price therefore reflects resale value alone, as in the pure-bubble environments of Tirole (1985) and Weil (1987).

Let $Z_t \in \{0, 1\}$ indicate whether the resale venue is active. Collapse is absorbing: if $Z_t = 0$, then $Z_{t+j} = 0$ for all $j \geq 1$. Conditional on $Z_t = 1$, let $B_t > 0$ be the price at which the young buy the token from the old. A holder who buys at date t receives B_{t+1} upon resale next period if the venue remains active, and zero otherwise. Sustaining the bubble creates no aggregate resources, and it organizes a transfer from the young to the old.

3.3 The Sponsor Partnership and Maintenance Technology

A predetermined share $s \in (0, 1]$ of each young cohort is designated as the sponsor partnership. The remaining share $1 - s$ consists of passive investors. Designation is imposed by the institution operating the market, not chosen by individual agents. All agents trade individually and anonymously. In the symmetric allocation derived below, each holds one token per capita, so s is also the partnership's share of aggregate token ownership.

After trade closes, the sponsor's holdings are locked and its manager chooses the survival probability²

$$q_t \equiv \Pr(Z_{t+1} = 1 \mid Z_t = 1) \in [0, 1).$$

The manager acts on behalf of the partnership. Its positions cannot be sold, hedged, or pledged before the survival outcome is realized. This restriction preserves the exposure that motivates maintenance. Throughout the baseline, retention is constant and publicly known.

Assumption 2 (Maintenance technology). The maintenance cost function

$$\psi : [0, 1) \rightarrow \mathbb{R}_+$$

is twice continuously differentiable and satisfies

$$\psi(0) = 0, \quad \psi'(0) \geq 0, \quad \psi''(\cdot) > 0, \quad \lim_{q \rightarrow 1} \psi'(q) = +\infty.$$

The function ψ is the cost of achieving survival probability q . Strict convexity captures increasing marginal difficulty in improving survival, and the divergence of ψ' at one rules out perfect survival at a finite marginal benefit. The condition $\psi'(0) \geq 0$ allows a positive marginal cost at zero maintenance.

²Appendix A derives this reduced-form choice from hidden operational effort.

The total maintenance cost does not depend on the sponsor's ownership share. It represents an infrastructure burden that must be borne regardless of how ownership is divided. Consequently, the sponsor pays the full cost but captures only the fraction s of holders' continuation benefit. Maintenance disutility is shared within the partnership through an allocation that does not depend on any individual member's token holdings. This assumption separates atomistic portfolio choices from the subsequent collective maintenance decision.

Maintenance is a utility cost. Aggregate goods feasibility is therefore

$$\int_0^1 c_{it}^y di + \int_0^1 c_{it}^o di = e_1 + e_2. \quad (2)$$

A goods-cost specification would additionally link maintenance to available consumption. We abstract from that resource channel to isolate the effect of delegated control and incomplete ownership retention.

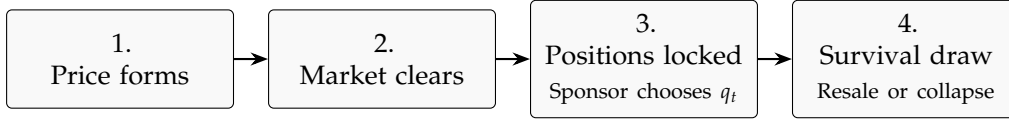
3.4 Timing

Conditional on $Z_t = 1$, events within date t follow the sequence in Figure 2.

1. The old hold the token, and a competitive resale price B_t is determined in equilibrium.
2. The market clears: the young acquire the unit supply at cost B_t , and the old receive B_t and exit.
3. Sponsor positions are locked. The sponsor chooses the hidden survival probability q_t .
4. Nature draws survival: with probability q_t the market remains live into $t + 1$; otherwise it collapses forever.

When maintenance is chosen, the current price has been paid and portfolios are fixed. The price B_t reflects anticipated upkeep but cannot be made contingent on the sponsor's

subsequent hidden action. The lock-up keeps the sponsor exposed to the next resale payoff during this decision. The old have already sold and exited, so maintenance is chosen for the benefit of current, not former, holders.



Hidden maintenance follows trade; the survival draw comes last

Figure 2: Within-period timing on a live date.

Note: Trading and clearing precede the hidden maintenance choice, and survival is realized afterward.

We restrict attention to rational-expectations equilibria with deterministic price sequences conditional on survival. At each live date, households correctly anticipate the sponsor's choice q_t and the next-period price B_{t+1} if the venue survives. The realized resale payment is B_{t+1} with probability q_t and zero otherwise. Prices and maintenance can vary over time, but there is no additional aggregate uncertainty or extrinsic randomization. Once the venue collapses, the token is permanently worthless.

4 Equilibrium Analysis

4.1 Household Pricing

A young agent who buys a_{it} units of the token at price $B_t > 0$ consumes $c_{it}^y = e_1 - B_t a_{it}$ when young. When old, the agent consumes

$$c_{i,t+1}^o = \begin{cases} e_2 + B_{t+1} a_{it}, & Z_{t+1} = 1, \\ e_2, & Z_{t+1} = 0. \end{cases}$$

Expected old-age consumption is therefore $e_2 + q_t B_{t+1} a_{it}$. Maintenance disutility, if any, is additive and independent of an individual agent's holdings, so it does not enter the

portfolio margin. Taking q_t and the live continuation price B_{t+1} as given, each young agent solves

$$\max_{0 \leq a_{it} < e_1/B_t} \{u(e_1 - B_t a_{it}) + \gamma(e_2 + q_t B_{t+1} a_{it})\}.$$

The objective is strictly concave. At an interior optimum,

$$B_t u'(e_1 - B_t a_{it}) = \gamma q_t B_{t+1}. \quad (3)$$

The left-hand side is the marginal utility cost of an additional token, and the right-hand side is its expected old-age benefit. The corner solution, i.e., the bubbleless equilibrium is treated separately below.

All young agents solve the same strictly concave portfolio problem. Token-market clearing, $\int_0^1 a_{it} di = 1$, therefore implies $a_{it} = 1$ for every agent in a symmetric positive-price equilibrium. The sponsor partnership consequently owns the share s of the unit supply. Substituting into the first-order condition gives

$$B_t u'(e_1 - B_t) = \gamma q_t B_{t+1}. \quad (4)$$

In a stationary live state $B_t = B_{t+1} = B > 0$, so (4) reduces to

$$u'(e_1 - B) = \gamma q. \quad (5)$$

Define the stationary survival threshold and the limiting stationary price

$$\underline{q} \equiv \frac{u'(e_1)}{\gamma} \in (0, 1), \quad \bar{B} \equiv e_1 - (u')^{-1}(\gamma) \in (0, e_1). \quad (6)$$

The stationary bubble price is given by

$$\mathcal{B}(q) \equiv e_1 - (u')^{-1}(\gamma q), \quad q \in (\underline{q}, 1). \quad (7)$$

Lemma 1 (Generalized Weil restriction). *Every positive stationary live path satisfies $q > \underline{q}$ and $B \in (0, \bar{B})$. The stationary bubble price $\mathcal{B}$ is continuous and strictly increasing.*

The proofs for the baseline model are collected in Appendix [D.1](#).

The lemma is Weil's restriction in general form. The threshold $\underline{q} = u'(e_1)/\gamma$ is the infimum of the survival probabilities consistent with a positive stationary price. At or below this threshold, no positive stationary price clears the token market. This restriction is stationary, since away from a steady state the household demand depends on both B_t and B_{t+1} . The monotonicity of $\mathcal{B}$ states that a higher stationary survival probability supports a higher stationary price.

Sponsor members satisfy the same portfolio first-order condition. Each member is atomistic and cannot individually affect the aggregate survival probability or the partnership's maintenance cost. The member therefore takes q_t as given at the trading stage, while the manager chooses maintenance collectively after trade. Rational expectations require the anticipated probability to equal that subsequent choice.

4.2 The Sponsor's Post-trade Problem

Once trade has closed, the price paid by the young is sunk and portfolio positions are fixed. At date t , the sponsor manager chooses the survival probability protecting the members' resale payoff at date $t + 1$. Aggregating over the sponsor partnership, the manager solves

$$\max_{q_t \in [0,1]} \{s\gamma q_t B_{t+1} - \psi(q_t)\}. \quad (8)$$

The objective is strictly concave because $\psi''(\cdot) > 0$. The first-order condition for an interior solution is

$$\psi'(q_t) = s\gamma B_{t+1}. \quad (9)$$

Allowing for a possible corner solution at zero maintenance, the sponsor best response is therefore

$$\mathbf{q}(B_{t+1}) \equiv \begin{cases} 0, & \text{if } s\gamma B_{t+1} \leq \psi'(0), \\ (\psi')^{-1}(s\gamma B_{t+1}), & \text{if } s\gamma B_{t+1} > \psi'(0). \end{cases} \quad (10)$$

The problem also admits a dual representation. Let ψ^* denote the Fenchel conjugate of ψ :

$$\psi^*(x) \equiv \sup_{q \in [0,1]} \{xq - \psi(q)\}.$$

The sponsor's optimized value from maintenance is therefore

$$V(B_{t+1}) = \psi^*(s\gamma B_{t+1}).$$

The sponsor's marginal value of survival is $s\gamma B_{t+1}$, whereas all current holders together value it at γB_{t+1} . Passive holders benefit from upkeep, but their gains do not enter the sponsor's objective. This creates a wedge between the valuation of the partnership versus all token holders. In appendix B, we analyze this governance externality.

Holding the continuation price fixed, a lower retained stake weakens maintenance incentives without changing the cost of upkeep. Maintenance falls weakly, and strictly when the optimum is interior. This is the internalization force familiar from retention and monitoring models (Chemla and Hennessy, 2014). At the limiting case $s = 0$, maintenance is zero at every continuation price, so a positive bubble cannot be sustained.

4.3 Stationary Equilibrium

Definition 1 (Live-path stationary equilibrium). A *live-path stationary equilibrium* is a pair (B, q) with stationary bubble price $B > 0$ and stationary survival probability $q \in (0, 1)$ such that

1. *Household optimality*: $B = \mathcal{B}(q)$;

2. *Sponsor optimality*: $q = \mathbf{q}(B)$;
3. *Market clearing*: each young agent holds one token per capita, and goods feasibility (2) holds.

A *bubbleless equilibrium* has $B = q = 0$, $c^y = e_1$, and $c^o = e_2$ at every date. A bubbleless equilibrium always exists, because a zero continuation price induces zero maintenance, and without a positive expected resale payoff no household pays a positive price. The question is whether the economy also admits a positive stationary equilibrium.

Theorem 1 (Existence and minimum retention). *Under Assumptions 1a and 2, a positive live-path stationary equilibrium exists if and only if*

$$s \geq s_{\min} \equiv \min_{q \in (0,1)} \frac{\psi'(q)}{\gamma[e_1 - (u')^{-1}(\gamma q)]}.$$

Some feasible retention level $s \in (0,1]$ can support a positive stationary price if and only if $s_{\min} \leq 1$.

The threshold $s_{\min}$ is the smallest stake that makes the sponsor willing to supply a survival probability supported by household demand. The numerator $\psi'(q)$ is marginal maintenance cost, and the denominator $\gamma\mathcal{B}(q)$ is the marginal value of survival to *all* holders at the associated stationary price. Their ratio is the ownership share needed to align the sponsor's private marginal benefit with that cost. Minimizing over q identifies the least retention consistent with a positive stationary bubble.

The following curvature restriction gives a sharper characterization.

Assumption 3 (Curvature). The marginal utility function u' and the marginal maintenance cost function ψ' are weakly convex.

Marginal utility being convex is satisfied by many standard utility functions (logarithmic, CRRA, CARA). Weak convexity of ψ' is the usual regularity condition on the

cost of raising a success probability in moral-hazard models with a continuum of effort (Laffont and Martimort, 2002).³ It holds for $\psi(q) \propto -\log(1-q)$, $\psi(q) \propto q/(1-q)$, and $\psi(q) \propto (1-q)^{-\eta} - 1$ with $\eta > 0$. These specifications also satisfy $\psi(0) = 0$ and $\lim_{q \rightarrow 1} \psi'(q) = +\infty$.

Proposition 1 (Multiplicity). *Suppose Assumptions 1a, 2, and 3 hold. If $s > s_{\min}$, there exist exactly two positive stationary equilibria.*

Write the two equilibria as (B_-, q_-) and (B_+, q_+) , ordered so that $B_- < B_+$ and $q_- < q_+$. In either equilibrium, beliefs about resale and maintenance incentives are mutually consistent. On the lower branch, a lower anticipated price gives the sponsor a weaker maintenance incentive, and the resulting lower survival probability supports that price. On the upper branch, a higher price and stronger maintenance reinforce one another. Both survival probabilities nevertheless exceed the household threshold $\underline{q}$, and the lower branch is not a zero-maintenance outcome.

4.4 Dynamics of the Bubble Price

We now study dynamic price paths conditional on survival. Household optimality and market clearing give (4). Along a positive live path, $q_t > 0$, so sponsor optimality is characterized by the interior condition (9). Eliminating B_{t+1} gives

$$B_t u'(e_1 - B_t) = \frac{q_t \psi'(q_t)}{s}. \quad (11)$$

The function $q \mapsto q\psi'(q)$ is continuous and strictly increasing from zero to infinity. Thus each current price $B_t \in (0, e_1)$ pins down a unique positive maintenance choice and an

³In (Laffont and Martimort, 2002, pp. 190–191), the agent chooses effort $e \in [0, 1]$ at cost $\psi(e)$ with $\psi', \psi'' > 0$ and $\lim_{e \rightarrow 1} \psi'(e) = +\infty$; $\psi''' \geq 0$ is imposed so that the information rent $e\psi'(e) - \psi(e)$ is convex and the principal's problem is concave. Our q is similar to the setup of Laffont and Martimort (2002).

associated continuation price,

$$B_{t+1} = T(B_t) \equiv \frac{\psi'(q_t)}{s\gamma}, \quad q_t \text{ solves (11).}$$

The map T is an algebraic candidate transition on $(0, e_1)$. A candidate continuation is *feasible* only if $T(B_t) < e_1$. The map is continuous and strictly increasing. When $s > s_{\min}$, its only positive fixed points are B_- and B_+ .

Theorem 2 (Dynamics of the bubble price). *Suppose Assumptions 1a, 2, and 3 hold and $s > s_{\min}$. Conditional on the market remaining live, if $B_0 \in (0, B_+)$, $B_t \rightarrow B_-$. If $B_0 \in (B_+, e_1)$, the resulting path is infeasible.*

The intuition is as follows. The household pricing condition can be written as

$$\frac{B_{t+1}}{B_t} = \frac{u'(e_1 - B_t)}{\gamma q_t}.$$

A higher current price absorbs more of the young cohort's endowment and raises the marginal utility cost of saving through the token. Supporting that price therefore requires a larger expected resale payoff. The sponsor can improve the probability of resale, but it does so only in response to the value of its locked stake, and at an increasing marginal cost. The price transition reconciles these two requirements, the payoff households need and the maintenance the sponsor is willing to supply.

At a stationary price B_i , differentiating household pricing after substituting the sponsor's best response gives

$$T'(B_i) = \frac{u'(e_1 - B_i) - B_i u''(e_1 - B_i)}{\gamma[\mathbf{q}(B_i) + B_i \mathbf{q}'(B_i)]}, \quad i \in \{-, +\}.$$

The numerator measures how a higher current price raises the marginal utility cost of an additional token, evaluated at market-clearing holdings. The denominator is the increase in its expected resale payoff when the continuation price rises, including the induced

maintenance response. At the lower equilibrium the latter response is stronger, so a price deviation requires a smaller continuation deviation and contracts over time. At the upper equilibrium the ranking is reversed, so deviations expand. Thus, the lower equilibrium is stable while the upper equilibrium is unstable.

The global movements follow the same logic. Below B_- , the implied resale opportunity is too weak to support a constant price, so households require appreciation conditional on survival. Between B_- and B_+ , maintenance supports sufficient expected resale value for the price to decline toward B_- . Above B_+ , continued appreciation would be required, but it cannot remain feasible. The next generation has only e_1 goods, and its marginal utility of the consumption left after buying the token diverges as the price approaches that bound.

Note that convergence along a live path does not imply permanent survival, because the venue can fail at any date through the survival draw. Moreover, the movements are equilibrium restrictions. In particular, an infeasible candidate sequence above B_+ is not an equilibrium boom that investors knowingly ride until an inevitable terminal crash. Rational expectations rule out its initial price. The theorem thus identifies the exact set of feasible prices.

Corollary 1 (Retention and the feasible price range). *Suppose Assumptions 1a, 2, and 3 hold. For each $s > s_{\min}$, the feasible price range is*

$$\mathcal{V}(s) = (0, B_+(s)].$$

If $s_{\min} < s' < s \leq 1$, then $\mathcal{V}(s') \subsetneq \mathcal{V}(s)$.

In practice, an unlock or vesting releases the maintenance provider's previously committed token exposure, leading to a reduction in retention s . Corollary 1 shows that a smaller retained stake tightens the price bound. At a fixed continuation price, a smaller stake lowers the sponsor's private return to maintenance without reducing its cost. The same force can be viewed from a fixed *current* price. Equation (11) then implies lower

maintenance when retention falls. Household pricing consequently requires a higher continuation price to compensate for the lower survival probability. A reduction in retention thus raises the future resale price needed to justify a given current payment, even as it lowers the largest price that can be sustained indefinitely. Prices near the old upper bound can cease to be feasible.

A stronger conclusion follows when retention becomes permanently smaller than $s_{\min}$. When that happens, positive stationary equilibria cease to exist and the increasing transition map has no positive fixed point. Every positive candidate price then requires further appreciation, and its continuation reaches the endowment bound after finitely many steps. Denote that date by T .

Forward-looking households cannot justify a current payment by a resale price that no future cohort can pay in equilibrium. If a zero resale value at date T is common knowledge, a young buyer at $T - 1$ receives no payoff from holding the token and therefore pays nothing. The same logic applies at $T - 2$ and recursively at every earlier date at which this news is known. Because the token has no dividend or other direct benefit, nothing remains to support its current valuation. The following result makes this announcement effect precise.

Theorem 3 (Unlock announcements and immediate bubble collapse). *Suppose Assumptions 1a and 2 hold. At a live date τ , before trade, it becomes common knowledge that there are a finite date $t_* \geq \tau$ and a constant $\hat{s} \in (0, 1]$ with $\hat{s} < s_{\min}$ such that the retained share satisfies*

$$0 \leq s_n \leq \hat{s} \quad \text{for every } n \geq t_*.$$

Then $B_\tau = 0$. Any positive valuation prevailing before the announcement collapses when the news arrives.

The result follows from the resale requirement, not from an assumption that maintenance immediately stops. Even below $s_{\min}$, a sufficiently high hypothetical continuation price can induce positive maintenance. But no sequence of such prices can satisfy both the sponsor's incentive constraint and investors' budget and optimality conditions indefinitely. Hence, conditional on reaching t_* , the token has no positive equilibrium resale value. A buyer at $t_* - 1$ then has nothing to resell, so that cohort also pays zero. The same reasoning applies at every earlier date at which the future regime is known, including τ .

This is an announcement effect, not a prediction of a fully anticipated crash on the mechanical unlock date. A new vesting agreement, a revised release schedule, or credible news that previously committed exposure will be withdrawn can reveal the loss of support before any tokens are sold. Conversely, if a schedule satisfying the theorem is already common knowledge when trading begins, a positive bubble cannot exist in the first place. The bound $s_n \leq \hat{s} < s_{\min}$ includes a one-time permanent reduction and subsequent rounds of further unlocking. A smaller release that leaves sufficient retention does not by itself imply a collapse to zero. Corollary 1 instead describes the contraction of the feasible price range.

Announcement-driven collapse differs from eventual stochastic failure. At a stationary live equilibrium, the probability of surviving another n dates is q^n , which tends to zero because $q < 1$. Collapse thus occurs almost surely in finite time even though the token

has a positive price after every surviving history (Weil, 1987; Blanchard and Watson, 1982). The same distinction applies to the nonstationary paths in Theorem 2: prices are bounded above by B_+ and one-period survival probabilities by $q_+ < 1$. Almost-sure eventual failure does not supply a known last trading date. In contrast, Theorem 3 supplies a known future date at which every equilibrium resale value is zero. Conditional price convergence, stochastic failure, and announcement-driven unraveling are therefore different equilibrium statements.

4.5 Token Unlocks and the Exposure of Insiders

The preceding results link insider retention to the sustainability of resale values. Corollary 1 shows that a lower retained share reduces the maximum sustainable price when retention remains above $s_{\min}$. Theorem 3 gives a sharper prediction: credible news of a permanent future reduction below the threshold eliminates positive resale values immediately. The mechanism operates through maintenance incentives, without requiring a change in total token supply or fundamental cash flows. The following episodes illustrate price movements ahead of unlocks and in response to changes in lock-up arrangements.

Prices can decline before previously locked tokens become transferable. On 21 October 2022, AXS was reported to have fallen more than 12.5 percent over the preceding 24 hours ahead of a late-October unlock.⁴ On 27 November 2023, DYDX, OP, and SUI fell ahead of unlocks scheduled to make almost \$650 million of previously locked tokens transferable that week. DYDX fell 7 percent ahead of a 1 December release of about \$480 million, reported as nearly doubling circulating supply. OP fell nearly 6 percent ahead of a 30 November unlock of about \$40 million, while SUI fell 8.8 percent with about \$48 million scheduled for release during the week. All three declines exceeded the concurrent 1.5

⁴Adam James, Axie Infinity Shards price drops sharply ahead of early investor token unlock, *The Block*, 21 October 2022, <https://www.theblock.co/news/web3/2022-10-21-axie-infinity-shards-price-drops-sharply-ahead-of-early-investor-token-unlock-178860>.

percent decline in the CoinDesk Market Index.⁵

An extension of transfer restrictions was accompanied by a price increase. In January 2023, the dYdX Foundation announced an extension of lock-ups on tokens issued to investors, employees, and consultants. The change postponed an initial release of 150 million tokens from 3 February 2023 to 1 December 2023, with subsequent monthly releases continuing through June 2026.⁶ On 25 January, DYDX rose as much as 21 percent as news of the delay became public.⁷ The price response thus preceded the originally scheduled release.

Admittedly, these patterns are not unique to the maintenance channel. An anticipated expansion in the tradable float can increase expected selling pressure. Investors who expect weaker future resale prices may sell in advance, putting downward pressure on the current price before the unlocked tokens reach the market. Conversely, postponing an unlock can raise the price by deferring expected sales. These responses can arise without any change in maintenance incentives, even when total token supply is fixed. Our model isolates a different channel: an unlock that reduces the maintainer's effective exposure weakens its incentive to preserve future resale. Transferability alone does not establish such a reduction, since insiders may continue to hold the tokens. The episodes therefore motivate the mechanism. Distinguishing the channels would require evidence on changes in the maintainer's unhedged exposure and maintenance activity, alongside expected sales and market liquidity.

⁵CoinDesk, Altcoin rally cools as massive \$650M worth of token unlocks loom over crypto market, 27 November 2023, <https://www.coindesk.com/markets/2023/11/27/altcoin-rally-cools-as-massive-650m-worth-of-token-unlocks-loom-over-crypto-market>.

⁶dYdX Foundation, Lock-up extension for \$DYDX issued to investors, employees and consultants, 24 January 2023, updated 31 January 2023, <https://www.dydx.foundation/blog/lock-up-extension>.

⁷Oliver Knight, DYDX token surges as unlock gets delayed until December, *CoinDesk*, 25 January 2023, <https://www.coindesk.com/business/2023/01/25/dydx-token-surges-21-as-report-says-token-unlock-delayed-until-december>.

5 Discussion

The benchmark isolates a governance component of resale value. Maintenance changes the probability of another trading opportunity, not the token's dividend or direct service payoff. Introducing redemption, transaction utility, or distributions from a productive platform would add a fundamental component. In that setting, a loss of maintenance incentives need not drive the entire token price to zero.

The relevant empirical state variable is the maintenance provider's effective, unhedged exposure, not the aggregate fraction of tokens reported as locked. An unlock for passive financial investors primarily changes their trading opportunities. An unlock that releases the controller's exposure can additionally change maintenance incentives. This distinction suggests comparing unlocks by recipient identity, residual insider holdings, alternative sources of operating finance, and subsequent measures of development or market support. The model predicts a tighter feasible price bound when committed exposure falls.

Retention is a commitment to exposure, not to a particular action or to successful survival. The baseline excludes separate compensation for maintenance, external subsidies, and franchise or reputational rents. These could sustain upkeep after token exposure declines, but their funding and enforcement would have to be modeled explicitly. Similarly, a persistent sponsor would introduce intertemporal incentives beyond the recurring post-trade problem studied here.

Longer lockups are therefore not an unconditional policy prescription. They preserve the exposure that motivates maintenance in this model, but can have costs or benefits through channels that the model omits. For example, liquidity can facilitate monitoring and informed blockholder trading ([Maug, 1998](#); [Edmans, 2009](#)). A theory of the optimal vesting schedule would need to trade off these forces and allow the sponsor's financing and trading decisions to respond. Our comparative statics hold the maintenance technology and the available incentive instruments fixed.

6 Conclusion

A pure bubble can depend on an institution that is costly to maintain. When the party responsible for that institution retains only part of the asset, its maintenance choice internalizes only part of the benefit from continued resale. Survival is then an equilibrium outcome of a governance problem.

The interaction between household demand and maintenance incentives produces a minimum-retention threshold for stationary existence. Under additional curvature restrictions, retention above that threshold supports two positive stationary equilibria with opposite comparative statics. Dynamic analysis distinguishes their roles. The lower price attracts all positive live paths below the upper price, while the upper price is the maximum feasible initial price. A larger retained stake expands this feasible interval.

The loss of governance support can also have an immediate valuation effect. A permanent reduction in retention below the threshold eliminates all positive continuation values and causes immediate backward unraveling. The model thus identifies a governance channel through which an unlock can affect valuation even without new issuance or a change in fundamental cash flows. The relevant question is not only how many tokens become tradable, but whether the party responsible for continued resale retains enough exposure to support it.

Internet Appendix

A An effort foundation for the survival choice

This appendix provides an effort-based foundation for the reduced-form maintenance technology in the main text. Maintenance is interpreted as hidden operational effort that raises the probability of continued resale. We show how an increasing, weakly concave survival technology and a strictly convex effort cost generate a strictly convex cost of achieving a given survival probability. Under the conditions stated below, this induced cost satisfies Assumption 2. The direct choice of survival can therefore be understood as a reparameterization of an underlying effort problem, preserving the sponsor's maintenance incentives.

Let the sponsor choose $e \geq 0$ at cost $C(e)$, and let survival be $q = \varphi(e)$, where $\varphi(0) = 0$, $\varphi'(e) > 0$, $\varphi''(e) \leq 0$, and $\varphi(e) \rightarrow 1$ as $e \rightarrow \infty$. Suppose also that $C(0) = 0$, $C'(e) \geq 0$, and $C''(e) > 0$. Because φ is strictly increasing, define

$$\psi(q) \equiv C(\varphi^{-1}(q)).$$

Choosing effort to maximize $s\gamma\varphi(e)B - C(e)$ is then equivalent to choosing survival to maximize the sponsor objective in (8). Writing $e = \varphi^{-1}(q)$, the derivatives of the reduced cost are

$$\begin{aligned}\psi'(q) &= \frac{C'(e)}{\varphi'(e)}, \\ \psi''(q) &= \frac{C''(e)\varphi'(e) - C'(e)\varphi''(e)}{[\varphi'(e)]^3} > 0.\end{aligned}$$

The final inequality follows from strictly convex effort cost and weakly concave survival production. The marginal-cost boundary condition in Assumption 2 follows whenever

$C'(e)/\varphi'(e) \rightarrow \infty$ as $e \rightarrow \infty$. If $C'(0) = 0$, then $\psi'(0) = 0$; if $C'(0) > 0$, then $\psi'(0) > 0$. Both cases are permitted by Assumption 2. Thus choosing q directly is a reduced-form representation of maintenance effort, not an assumption that the sponsor literally purchases a probability.

B The governance externality

This appendix examines the governance externality created by incomplete ownership retention. The sponsor bears the full cost of maintenance, while part of the resulting benefit accrues to passive token holders. Holding the continuation price and post-trade portfolios fixed, we compare the sponsor's choice with the maintenance decision that maximizes the joint continuation value of all current holders. This comparison shows why delegated control can lead to insufficient maintenance and how greater retention narrows the incentive wedge.

Fix a continuation price B and the post-trade portfolio positions. The sponsor chooses maintenance according to

$$V^I(q; B) \equiv s\gamma qB - \psi(q),$$

where the superscript I denotes the decentralized institution in which the sponsor internalizes only its retained stake.

As a benchmark, consider the maintenance choice that would arise if all current token holders jointly controlled the decision. The corresponding objective is

$$V^A(q; B) \equiv \gamma qB - \psi(q),$$

where the superscript A denotes the all-owner benchmark. This is not a social welfare function. It is the joint continuation value of current token holders at the post-trade stage, taking the price already paid and the future resale value as given.

The difference between the two problems is the value of the outside float. Differentiating the two objectives gives

$$\frac{\partial V^A(q; B)}{\partial q} - \frac{\partial V^I(q; B)}{\partial q} = (1 - s)\gamma B \geq 0.$$

The inequality is strict when $s < 1$ and $B > 0$. The wedge is the marginal continuation value generated by the passive holdings. The sponsor protects the resale value of the entire token, but it captures only the fraction s that it retains. Because the maintenance cost is independent of ownership concentration, the sponsor bears the full cost while internalizing only part of the benefit.

Define the all-owner maintenance choice as

$$q^A(B) \equiv \arg \max_{q \in [0,1]} \{\gamma q B - \psi(q)\}.$$

The corresponding value function is

$$V^A(B) = \psi^*(\gamma B).$$

The decentralized sponsor chooses

$$\mathbf{q}(B) \equiv \arg \max_{q \in [0,1]} \{s\gamma q B - \psi(q)\},$$

with value

$$V^I(B) = \psi^*(s\gamma B).$$

Proposition 2 (Governance wedge). *Fix a continuation price $B > 0$. If $s < 1$, then*

$$\mathbf{q}(B) \leq q^A(B)$$

and

$$V^A(B) - V^I(B) = \psi^*(\gamma B) - \psi^*(s\gamma B) \geq 0.$$

Both inequalities are strict whenever $\gamma B > \psi'(0)$. In particular, if $\psi'(0) = 0$, they are strict for every $B > 0$.

The proposition isolates the governance distortion created by incomplete ownership retention. The sponsor controls maintenance but does not internalize the value accruing to the outside float. In primal terms, once the all-owner maintenance margin is active, the distortion appears as insufficient maintenance,

$$\mathbf{q}(B) < q^A(B);$$

in dual terms, it appears as the value gap

$$\psi^*(\gamma B) - \psi^*(s\gamma B).$$

A larger retained stake narrows the incentive wedge and moves the maintenance choice toward the all-owner benchmark, without creating a fundamental payoff for the token. The two optimized value functions use different ownership weights. Their difference is therefore not, by itself, a measure of deadweight loss or a welfare comparison across generations.

C Nonlinear old-age utility

This appendix allows nonlinear old-age utility while retaining the baseline endowments and timing. Assumption 1b replaces Assumption 1a; the maintenance technology in Assumption 2 is unchanged. The symbols $\underline{q}$, $\bar{B}$, $\mathcal{B}$, and $s_{\min}$ are redefined below for this extension. Its proofs are collected in Appendix D.3.

Each generation is endowed with $e_1 > 0$ units of the consumption good when young and $e_2 \geq 0$ units when old. Preferences are

$$u(c_{it}^y) + \gamma E_t[v(c_{i,t+1}^o)] - \text{maintenance disutility, if any,}$$

where $\gamma > 0$. As in the baseline, we consider deterministic price sequences conditional on survival. The survival realization is the only aggregate uncertainty.

Assumption 1b (Preferences). The function $u : (0, e_1] \rightarrow \mathbb{R}$ is twice continuously differentiable and satisfies

$$u' > 0, \quad u'' < 0, \quad \lim_{c \downarrow 0} u'(c) = +\infty.$$

The function $v : [e_2, \infty) \rightarrow \mathbb{R}$ is continuously differentiable, with a right derivative at e_2 , and is twice continuously differentiable on (e_2, ∞) . It satisfies

$$v' > 0, \quad v'' \leq 0.$$

In particular, $v(e_2)$ and the right derivative $v'(e_2)$ are finite. Moreover,

$$u'(e_1) < \gamma v'(e_2). \tag{C.1}$$

Condition (C.1) is the generalized dynamic-inefficiency restriction. Strict concavity of u ensures unique individual demand and a strictly increasing stationary bubble price.

C.1 Household demand

At a live date t , a young agent purchasing a_{it} tokens consumes

$$c_{it}^y = e_1 - B_t a_{it}, \quad c_{i,t+1}^o = \begin{cases} e_2 + B_{t+1} a_{it}, & Z_{t+1} = 1, \\ e_2, & Z_{t+1} = 0. \end{cases}$$

Taking prices and the anticipated survival probability as given, the agent solves

$$\max_{0 \leq a_{it} < e_1/B_t} \{u(e_1 - B_t a_{it}) + \gamma [q_t v(e_2 + B_{t+1} a_{it}) + (1 - q_t) v(e_2)]\}.$$

As in the baseline, maintenance disutility is additive and does not change with an individual atomistic agent's portfolio choice, so it does not enter this problem.

The objective is strictly concave. At an interior optimum,

$$B_t u'(e_1 - B_t a_{it}) = \gamma q_t B_{t+1} v'(e_2 + B_{t+1} a_{it}).$$

Symmetry and market clearing give $a_{it} = 1$, and therefore

$$B_t u'(e_1 - B_t) = \gamma q_t B_{t+1} v'(e_2 + B_{t+1}).$$

A positive live price must satisfy $B_t < e_1$. Aggregate consumption at a live date is $(e_1 - B_t) + (e_2 + B_t) = e_1 + e_2$, so token payments remain transfers between generations.

C.2 The stationary bubble price

In a stationary live state, $B_t = B_{t+1} = B > 0$, and household optimality becomes

$$u'(e_1 - B) = \gamma q v'(e_2 + B). \tag{C.2}$$

Define

$$\underline{q} \equiv \frac{u'(e_1)}{\gamma v'(e_2)} \in (0, 1),$$

and let $\bar{B} \in (0, e_1)$ solve

$$u'(e_1 - \bar{B}) = \gamma v'(e_2 + \bar{B}).$$

For $q \in (\underline{q}, 1)$, denote the solution to (C.2) by $\mathcal{B}(q)$.

Lemma 2 (Generalized Weil restriction). *For a given $q \in [0, 1)$, a positive stationary household price exists if and only if $q > \underline{q}$. It is uniquely given by $B = \mathcal{B}(q) \in (0, \bar{B})$. The bubble price is continuously differentiable and strictly increasing, with*

$$\mathcal{B}'(q) = \frac{\gamma v'(e_2 + \mathcal{B}(q))}{-u''(e_1 - \mathcal{B}(q)) - \gamma q v''(e_2 + \mathcal{B}(q))} > 0, \quad (\text{C.3})$$

and endpoint limits

$$\lim_{q \downarrow \underline{q}} \mathcal{B}(q) = 0, \quad \lim_{q \uparrow 1} \mathcal{B}(q) = \bar{B}.$$

The threshold $\underline{q}$ is the infimum of the survival probabilities consistent with a positive stationary price. The inequality must be strict. The bound $\bar{B}$ is the limiting stationary price as survival tends to certainty. These are stationary restrictions, not bounds on all nonstationary prices or survival probabilities.

C.3 The sponsor's post-trade problem

For a continuation price $B \in [0, e_1]$, define the survival utility gain

$$W(B) \equiv v(e_2 + B) - v(e_2).$$

The manager maximizes the aggregate utility of the mass s of sponsor members. Omitting terms independent of maintenance, the date- t problem is

$$\max_{q \in [0, 1)} \{s\gamma q W(B_{t+1}) - \psi(q)\}.$$

Thus the best response to a continuation price B is

$$\mathbf{q}(B) = \begin{cases} 0, & s\gamma W(B) \leq \psi'(0), \\ (\psi')^{-1}(s\gamma W(B)), & s\gamma W(B) > \psi'(0). \end{cases}$$

This function is continuous and weakly increasing. Wherever $\mathbf{q}(B) > 0$, it is continuously differentiable, with

$$\mathbf{q}'(B) = \frac{s\gamma v'(e_2 + B)}{\psi''(\mathbf{q}(B))} > 0. \quad (\text{C.4})$$

Along a positive live path, household pricing requires $q_t > 0$. The sponsor's choice is therefore interior and satisfies

$$q_t = \mathbf{q}(B_{t+1}), \quad \psi'(q_t) = s\gamma W(B_{t+1}).$$

C.4 Stationary equilibrium

Theorem 4 (Existence and minimum retention). *Under Assumptions 1b and 2, a positive stationary live equilibrium exists if and only if $s \geq s_{\min}$, where*

$$s_{\min} \equiv \min_{q \in (q, 1)} \frac{\psi'(q)}{\gamma W(\mathcal{B}(q))} > 0.$$

A feasible retention level $s \in (0, 1]$ supports a positive stationary price if and only if $s_{\min} \leq 1$.

The threshold compares marginal maintenance cost with the utility value of survival supported by household demand. The bubbleless stationary equilibrium also persists: if the continuation price is zero, the sponsor chooses zero maintenance, and no household is willing to pay a positive current price.

The following restriction replaces the baseline curvature condition in Assumption 3 and adds a sufficient condition for a single-valued, increasing dynamic price transition.

Assumption 5 (Curvature and transition monotonicity). The marginal maintenance cost ψ' is weakly convex. In addition,

$$q \mapsto W(\mathcal{B}(q)) \quad \text{is weakly concave on } (\underline{q}, 1), \quad (\text{C.5})$$

and

$$v'(e_2 + B) + Bv''(e_2 + B) \geq 0, \quad B \in (0, e_1]. \quad (\text{C.6})$$

Condition (C.6) is a sufficient condition for a monotone price transition. It is not needed for stationary existence or multiplicity.⁸

When $v(c) = c$, condition (C.6) is automatic and the stationary bubble price becomes

$$\mathcal{B}(q) = e_1 - (u')^{-1}(\gamma q).$$

In this case, convexity of u' implies (C.5), recovering the linear-old-age-utility assumptions.

Proposition 3 (Multiplicity). *Under Assumptions 1b, 2, and 5, if $s > s_{\min}$, there are exactly two positive stationary equilibria (B_-, q_-) and (B_+, q_+) , with*

$$\underline{q} < q_- < q_+ < 1, \quad 0 < B_- < B_+ < \bar{B}.$$

C.5 Dynamics of the bubble price

Substituting the sponsor best response into household pricing gives

$$B_t u'(e_1 - B_t) = \gamma \mathbf{q}(B_{t+1}) B_{t+1} v'(e_2 + B_{t+1}). \quad (\text{C.7})$$

⁸Condition (C.5) can also be checked directly from preferences: it is equivalent to requiring

$$B \mapsto \frac{-u''(e_1 - B)v'(e_2 + B) - u'(e_1 - B)v''(e_2 + B)}{[v'(e_2 + B)]^3}$$

to be weakly increasing on $(0, \bar{B})$. This equivalence follows by differentiating $W(\mathcal{B}(q))$ and using the stationary pricing condition. It requires only the maintained second derivatives, not third derivatives.

For each current price admitting a solution $B_{t+1} \in (0, e_1)$, this equation defines the live-price transition

$$B_{t+1} = T(B_t).$$

The map is defined only where a positive feasible continuation exists. No extension beyond that domain is required.

Theorem 5 (Dynamics of the bubble price). *Suppose Assumptions 1b, 2, and 5 hold, and $s > s_{\min}$. Conditional on survival, every $B_0 \in (0, B_+)$ generates a feasible price path converging monotonically to B_- . The price $B_0 = B_+$ generates the upper stationary path. For $B_0 \in (B_+, e_1)$, a positive feasible continuation either fails to exist immediately or fails after finitely many iterations. The feasible positive initial prices are therefore exactly $(0, B_+]$.*

The zero-price equilibrium is treated separately from the positive-price transition. In particular, the proof does not assume $T(0) = 0$ or $\psi'(0) = 0$.

D Proofs

D.1 The baseline model

The following proofs use the baseline model with linear old-age utility.

We record some properties of the baseline best response $\mathbf{q}$, including its zero-maintenance corner. In the following lemma, B denotes the continuation price, and the dependence of $\mathbf{q}$ on s is suppressed in the notation.

Lemma 3 (Properties of the sponsor best response). *The sponsor best response defined in (10) satisfies*

$$\mathbf{q}(0) = 0, \quad \lim_{s \rightarrow 0} \mathbf{q}(B) = 0, \quad \mathbf{q}(B) < 1, \text{ for any finite } B.$$

Moreover, whenever the solution is interior,

$$\frac{\partial \mathbf{q}(B)}{\partial B} > 0, \quad \frac{\partial \mathbf{q}(B)}{\partial s} > 0.$$

Proof of Lemma 1. Because u' is continuous and strictly decreasing, the stationary condition (5) has a positive-price solution if and only if $\gamma q > u'(e_1)$, that is $q > \underline{q}$. The Inada condition then gives the unique solution (7). Since $(u')^{-1}$ is strictly decreasing, $\mathcal{B}$ is strictly increasing. Thus, the endpoint limits follow from (6). $\square$

Proof of Theorem 1. A positive stationary equilibrium requires $B = \mathcal{B}(q)$ and $\psi'(q) = s\gamma\mathcal{B}(q)$, or equivalently $s = h(q)$ where

$$h(q) \equiv \frac{\psi'(q)}{\gamma[e_1 - (u')^{-1}(\gamma q)]}, \quad q \in (\underline{q}, 1).$$

The function h is continuous. As $q \downarrow \underline{q}$ the denominator converges to zero from above while $\psi'(q) \rightarrow \psi'(\underline{q}) > 0$, so $h(q) \rightarrow +\infty$. As $q \uparrow 1$ the denominator converges to the finite positive number $\gamma\bar{B}$ while $\psi'(q) \rightarrow +\infty$, so again $h(q) \rightarrow +\infty$. Hence h attains a strictly

positive minimum at an interior point, and a solution to $h(q) = s$ exists if and only if s is weakly above this minimum. The feasibility statement follows because s is a share and therefore cannot exceed one. $\square$

Proof of Proposition 1. Since u' is decreasing and convex, its inverse is decreasing and convex. Hence $\mathcal{B}(q) = e_1 - (u')^{-1}(\gamma q)$ is concave under Assumption 3. Define the *equilibrium gap*

$$G(q; s) \equiv s\gamma\mathcal{B}(q) - \psi'(q), \quad q \in (\underline{q}, 1). \quad (\text{D.1})$$

A positive stationary equilibrium is exactly a zero of $G(\cdot; s)$. Since ψ' is convex, $-\psi'$ is concave, and therefore $G(\cdot; s)$ is concave. Moreover

$$\lim_{q \downarrow \underline{q}} G(q; s) = -\psi'(\underline{q}) < 0, \quad \lim_{q \uparrow 1} G(q; s) = -\infty.$$

When $s > s_{\min}$, G is strictly positive at some interior point. Continuity gives one zero on each side of the positive region. Concavity implies that the nonnegative set of G is an interval and rules out any additional crossing. Hence there are exactly two zeros $q_- < q_+$, with

$$G(q; s) \begin{cases} < 0, & q \in (\underline{q}, q_-), \\ > 0, & q \in (q_-, q_+), \\ < 0, & q \in (q_+, 1). \end{cases}$$

Concavity and strict positivity between the roots also imply

$$G_q(q_-; s) > 0 > G_q(q_+; s).$$

Since $\mathcal{B}$ is strictly increasing, $B_- < B_+$. $\square$

The same geometry delivers branch-specific comparative statics.

Lemma 4 (Comparative statics). *Suppose Assumptions 1a, 2, and 3 hold and $s > s_{\min}$.*

$$\frac{\partial B_+}{\partial s} > 0, \quad \frac{\partial B_-}{\partial s} < 0, \quad \frac{\partial B_+}{\partial \gamma} > 0, \quad \frac{\partial B_-}{\partial \gamma} < 0.$$

The same signs apply to the corresponding survival probabilities $q_{\pm}$.

Greater retention moves the stationary equilibria in opposite directions. On the upper branch, price and survival increase. On the lower branch, both decrease. A larger old-age utility weight γ has the same sign pattern.

Proof of Lemma 4. Making the dependence on γ explicit, the equilibrium gap in (D.1) is

$$G(q; s, \gamma) = s\gamma\mathcal{B}(q; \gamma) - \psi'(q), \quad \mathcal{B}(q; \gamma) = e_1 - (u')^{-1}(\gamma q).$$

The proof of Proposition 1 gives $G_q(q_-; s, \gamma) > 0 > G_q(q_+; s, \gamma)$. At either equilibrium, implicit differentiation therefore yields

$$\frac{\partial q_i}{\partial s} = -\frac{G_s(q_i; s, \gamma)}{G_q(q_i; s, \gamma)}, \quad \frac{\partial q_i}{\partial \gamma} = -\frac{G_\gamma(q_i; s, \gamma)}{G_q(q_i; s, \gamma)}, \quad i \in \{-, +\}.$$

Both partial derivatives in the numerators are positive, since

$$G_s = \gamma\mathcal{B} > 0, \quad G_\gamma = s \left[\mathcal{B} + \gamma \frac{\partial \mathcal{B}}{\partial \gamma} \right] > 0, \quad \frac{\partial \mathcal{B}}{\partial \gamma} = -\frac{q}{u''(e_1 - \mathcal{B})} > 0.$$

This establishes the signs of both comparative statics on each survival branch.

Because $\mathcal{B}$ has no direct dependence on s and $\mathcal{B}_q = -\gamma/u'' > 0$, the signs of $\partial B_i/\partial s$ coincide with those of $\partial q_i/\partial s$. For γ , the direct effect on household pricing must also be included. Using the preceding derivatives and $G_q = s\gamma\mathcal{B}_q - \psi''$, we obtain

$$\begin{aligned} \frac{\partial B_i}{\partial \gamma} &= \frac{\partial \mathcal{B}}{\partial \gamma}(q_i; \gamma) + \frac{\partial \mathcal{B}}{\partial q}(q_i; \gamma) \frac{\partial q_i}{\partial \gamma} \\ &= \frac{s\gamma\mathcal{B}_i + q_i\psi''(q_i)}{u''(e_1 - B_i)G_q(q_i; s, \gamma)}. \end{aligned}$$

The numerator is positive and $u'' < 0$, so this derivative has the opposite sign to $G_q(q_i; s, \gamma)$. Thus $\partial B_+ / \partial \gamma > 0$ and $\partial B_- / \partial \gamma < 0$, as claimed. $\square$

Proof of Theorem 2. For each current price $B \in (0, e_1)$, the associated maintenance choice $q \in (0, 1)$ and the continuation price satisfy

$$q\psi'(q) = sBu'(e_1 - B), \quad T(B) = \frac{\psi'(q)}{s\gamma}.$$

Both $B \mapsto Bu'(e_1 - B)$ and $q \mapsto q\psi'(q)$ are continuous and strictly increasing, with range $(0, \infty)$. Thus q is uniquely determined and strictly increasing in B . Implicit differentiation gives

$$T'(B) = \frac{\psi''(q)[u'(e_1 - B) - Bu''(e_1 - B)]}{\gamma[\psi'(q) + q\psi''(q)]} > 0.$$

Hence T is continuously differentiable and strictly increasing on $(0, e_1)$.

A positive fixed point $T(B) = B$ satisfies

$$\psi'(q) = s\gamma B, \quad u'(e_1 - B) = \gamma q,$$

which are exactly the stationary equilibrium conditions. Conversely, every positive stationary equilibrium is a fixed point. Proposition 1 therefore implies that the only positive fixed points are B_- and B_+ .

We next determine the direction of motion. At either stationary equilibrium (B_i, q_i) , $i \in \{-, +\}$, differentiating the equilibrium gap in (D.1) gives

$$G_q(q_i; s) = -\frac{s\gamma^2}{u''(e_1 - B_i)} - \psi''(q_i).$$

Using the stationary conditions in the expression for T' yields

$$T'(B_i) - 1 = \frac{B_i u''(e_1 - B_i)}{\gamma[\psi'(q_i) + q_i \psi''(q_i)]} G_q(q_i; s).$$

The coefficient multiplying G_q is strictly negative. By Proposition 1, G is concave and strictly positive between its two zeros, so

$$G_q(q_-; s) > 0 > G_q(q_+; s).$$

Consequently,

$$0 < T'(B_-) < 1 < T'(B_+).$$

These slope inequalities determine the sign of $T(B) - B$ near each fixed point. Since this difference is continuous and has no other zeros, its sign cannot change elsewhere within any of the three intervals. Therefore,

$$T(B) - B \begin{cases} > 0, & B \in (0, B_-), \\ < 0, & B \in (B_-, B_+), \\ > 0, & B \in (B_+, e_1). \end{cases}$$

Suppose first that $B_0 \in (0, B_-)$. The sign pattern and strict monotonicity of T imply

$$B_t < B_{t+1} < B_-$$

at every date along the live path. The sequence is therefore increasing and bounded above by B_- . Its limit is positive and, by continuity, must be a fixed point. Hence $B_t \uparrow B_-$.

Similarly, if $B_0 \in (B_-, B_+)$, then

$$B_- < B_{t+1} < B_t,$$

so the sequence decreases to a fixed point in $[B_-, B_0]$. The only such fixed point is B_- , and therefore $B_t \downarrow B_-$. The cases $B_0 = B_-$ and $B_0 = B_+$ yield stationary paths.

Finally, suppose $B_0 \in (B_+, e_1)$. The implied price sequence is strictly increasing for as

long as it remains feasible. Suppose, toward a contradiction, that $B_t < e_1$ at every date. Then B_t converges to some $\ell \in (B_+, e_1]$. If $\ell < e_1$, continuity implies

$$T(\ell) = \ell,$$

contradicting the absence of a fixed point above B_+ . If $\ell = e_1$, the household pricing condition implies

$$B_t u'(e_1 - B_t) = \gamma q_t B_{t+1} < \gamma e_1$$

at every date. This contradicts $u'(e_1 - B_t) \rightarrow +\infty$ as $B_t \uparrow e_1$.

Thus the implied continuation price must reach or exceed e_1 after finitely many iterations, violating feasibility. No initial price above B_+ can therefore be part of an infinite-horizon positive live equilibrium. $\square$

Proof of Corollary 1. Fix $s > s_{\min}$. By Theorem 2, every initial price in $(0, B_+(s))$ has a feasible path converging to $B_-(s)$, and $B_+(s)$ has the upper stationary continuation. Every larger initial price eventually violates feasibility. Hence $\mathcal{V}(s) = (0, B_+(s)]$. Lemma 4 gives $\partial B_+ / \partial s > 0$. Therefore $s_{\min} < s' < s \leq 1$ implies

$$(0, B_+(s')] \subsetneq (0, B_+(s)],$$

as claimed. $\square$

Proof of Theorem 3. Write q_r for the maintenance choice associated with a current price $B \in (0, e_1)$ and a positive retained share r , defined by

$$q_r \psi'(q_r) = r B u'(e_1 - B).$$

The dependence of q_r on the fixed current price B is suppressed. Because $q \mapsto q \psi'(q)$ is continuous and strictly increasing from zero to infinity, $q_r \in (0, 1)$ is uniquely defined and

strictly increasing in r . The candidate continuation is

$$T_r(B) = \frac{\psi'(q_r)}{r\gamma} = \frac{Bu'(e_1 - B)}{\gamma q_r}.$$

Thus $T_r(B)$ is strictly decreasing in r . In particular, for $0 < r \leq \hat{s}$,

$$T_r(B) \geq T_{\hat{s}}(B).$$

The map $T_{\hat{s}}$ is continuous on $(0, e_1)$ and tends to infinity as $B \uparrow e_1$. A positive fixed point would be a stationary equilibrium with retention $\hat{s}$, which is impossible by Theorem 1 because $\hat{s} < s_{\min}$. Continuity and the behavior near e_1 therefore imply

$$T_{\hat{s}}(B) > B \quad \text{for every } B \in (0, e_1).$$

Suppose a positive equilibrium price could be sustained at date t_* . Household pricing requires a positive survival probability and a positive resale value at every subsequent live date. If $s_n = 0$ at any such date, the sponsor chooses zero maintenance, immediately contradicting this requirement. Otherwise $s_n \in (0, \hat{s}]$ for every $n \geq t_*$, and equilibrium requires

$$B_{n+1} = T_{s_n}(B_n) \geq T_{\hat{s}}(B_n) > B_n.$$

The positive price sequence is therefore strictly increasing. Household pricing and feasibility also imply

$$B_n u'(e_1 - B_n) = \gamma q_n B_{n+1} < \gamma e_1.$$

The Inada condition bounds B_n away from e_1 , so the sequence converges to a limit $\ell \in (0, e_1)$. Passing to the limit in the preceding transition inequality gives

$$\ell \geq T_{\hat{s}}(\ell) > \ell,$$

a contradiction. There is no positive infinite-horizon continuation from date t_* , and hence $B_{t_*} = 0$ on any history reaching that date.

At date τ , this absence of a positive continuation is common knowledge. If $t_* > \tau$, a positive price at $t_* - 1$ would require a positive resale value at t_* , which has just been ruled out. Repeating the argument backward yields

$$B_n = 0, \quad n = \tau, \dots, t_*.$$

In particular, $B_\tau = 0$. This conclusion concerns prices after the announcement; it does not alter payments made before the information became available. $\square$

D.2 Additional properties of the sponsor best response

Proof of Lemma 3. For a fixed continuation price $B \geq 0$, the sponsor's objective has first derivative $s\gamma B - \psi'(q)$ and second derivative $-\psi''(q) < 0$. If $s\gamma B \leq \psi'(0)$, its derivative is nonpositive at zero and strictly negative thereafter, so zero is the unique maximizer. Otherwise, the derivative is positive at zero and tends to $-\infty$ as $q \uparrow 1$, giving the unique interior solution in (10). In particular, $\mathbf{q}(0) = 0$, and every finite B gives $\mathbf{q}(B) < 1$.

As $s \downarrow 0$, the marginal benefit $s\gamma B$ tends to zero. If $\psi'(0) > 0$, the choice is eventually the zero corner. If $\psi'(0) = 0$, continuity and strict monotonicity of ψ' imply $(\psi')^{-1}(s\gamma B) \rightarrow 0$. Hence $\mathbf{q}(B) \rightarrow 0$ in either case.

At an interior solution, implicit differentiation of $\psi'(\mathbf{q}(B)) = s\gamma B$ gives

$$\frac{\partial \mathbf{q}(B)}{\partial B} = \frac{s\gamma}{\psi''(\mathbf{q}(B))} > 0, \quad \frac{\partial \mathbf{q}(B)}{\partial s} = \frac{\gamma B}{\psi''(\mathbf{q}(B))} > 0.$$

This proves the stated properties. $\square$

D.3 Nonlinear old-age utility

All objects in this subsection are those of Appendix C.

Proof of Lemma 2. For a fixed $q \geq 0$, the difference

$$u'(e_1 - B) - \gamma q v'(e_2 + B)$$

is strictly increasing in B and diverges to $+\infty$ as $B \uparrow e_1$. Its value at $B = 0$ is negative exactly when $q > \underline{q}$. This proves existence and uniqueness, including the existence of $\bar{B}$ by setting $q = 1$. Implicit differentiation gives (C.3). Monotonicity and continuity give the endpoint limits. $\square$

Proof of Theorem 4. A positive stationary equilibrium satisfies $B = \mathcal{B}(q)$ and $q = \mathbf{q}(B)$. Equivalently, it is a zero of the equilibrium gap

$$G(q; s) \equiv s\gamma W(\mathcal{B}(q)) - \psi'(q), \quad q \in (\underline{q}, 1). \quad (\text{D.2})$$

The stationary conditions are equivalent to

$$s = \frac{\psi'(q)}{\gamma W(\mathcal{B}(q))}.$$

The ratio is positive and continuous on $(\underline{q}, 1)$. As $q \downarrow \underline{q}$, its denominator tends to zero while its numerator tends to $\psi'(\underline{q}) > 0$. As $q \uparrow 1$, its denominator tends to the finite positive value $\gamma W(\bar{B})$ while its numerator diverges. Hence the ratio diverges at both endpoints, attains a positive minimum in the interior, and has range $[s_{\min}, \infty)$. This proves both claims. $\square$

Proof of Proposition 3. The gap G defined in (D.2) is concave, with

$$\lim_{q \downarrow \underline{q}} G(q; s) = -\psi'(\underline{q}) < 0, \quad \lim_{q \uparrow 1} G(q; s) = -\infty.$$

Since $s > s_{\min}$, it is strictly positive at an interior point. Continuity gives a zero on either side of that point, and concavity rules out additional zeros. Thus

$$G(q; s) \begin{cases} < 0, & q \in (\underline{q}, q_-), \\ > 0, & q \in (q_-, q_+), \\ < 0, & q \in (q_+, 1). \end{cases}$$

Moreover, concavity and strict positivity between the roots imply

$$G_q(q_-; s) > 0 > G_q(q_+; s). \quad (\text{D.3})$$

The ordering of prices follows from $\mathcal{B}' > 0$. □

Proof of Theorem 5. First consider the right-hand side of (C.7) as a function of the continuation price x . It is continuous, equals zero at $x = 0$, and is strictly increasing wherever $\mathbf{q}(x) > 0$, because

$$\begin{aligned} \frac{d}{dx} [\gamma \mathbf{q}(x) x v'(e_2 + x)] &= \gamma \mathbf{q}'(x) x v'(e_2 + x) \\ &\quad + \gamma \mathbf{q}(x) [v'(e_2 + x) + x v''(e_2 + x)] > 0. \end{aligned}$$

Here the first term is strictly positive and the second is nonnegative. By contrast, the current-price expression satisfies

$$\frac{d}{dB} [B u'(e_1 - B)] = u'(e_1 - B) - B u''(e_1 - B) > 0,$$

and increases from zero to infinity on $[0, e_1)$. Consequently, current prices admitting a positive feasible continuation form an interval, and T is uniquely defined, continuously differentiable, and strictly increasing there. This interval contains $(0, B_+]$ and some prices above B_+ : B_+ is a feasible stationary price, and the continuation-value expression is strictly

increasing in a neighborhood of $x = B_+$.

A positive fixed point satisfies

$$u'(e_1 - B) = \gamma q v'(e_2 + B), \quad q = \mathbf{q}(B) > 0.$$

These are precisely the stationary equilibrium conditions. Thus Proposition 3 implies that B_- and B_+ are the only positive fixed points.

We next establish local stability and use it to determine the global direction of motion. At a fixed point (B_i, q_i) , $i \in \{-, +\}$, implicit differentiation gives

$$\begin{aligned} u'(e_1 - B_i) - B_i u''(e_1 - B_i) &= \gamma T'(B_i) \left\{ B_i v'(e_2 + B_i) \mathbf{q}'(B_i) \right. \\ &\quad \left. + q_i [v'(e_2 + B_i) + B_i v''(e_2 + B_i)] \right\}. \end{aligned}$$

The coefficient of $T'(B_i)$ is strictly positive. Subtracting that coefficient from the left-hand side, and using stationary pricing together with (C.3), yields

$$\text{sgn}[T'(B_i) - 1] = \text{sgn} \left[\frac{1}{\mathcal{B}'(q_i)} - \mathbf{q}'(B_i) \right].$$

Meanwhile, differentiating (D.2) and using (C.4) gives

$$G_q(q_i; s) = \psi''(q_i) [\mathbf{q}'(B_i) \mathcal{B}'(q_i) - 1].$$

Since $\mathcal{B}' > 0$ and $\psi'' > 0$, it follows that

$$\text{sgn}[T'(B_i) - 1] = -\text{sgn } G_q(q_i; s).$$

Together with (D.3) and $T' > 0$, this proves

$$0 < T'(B_-) < 1 < T'(B_+).$$

These slopes determine the sign of $T(B) - B$ near the two fixed points. Since the difference is continuous and has no other zeros on its interval of definition, its sign cannot change elsewhere within the corresponding intervals. Hence

$$T(B) - B \begin{cases} > 0, & 0 < B < B_-, \\ < 0, & B_- < B < B_+, \\ > 0, & B > B_+, \end{cases} \quad (\text{D.4})$$

where the last line applies only when a feasible continuation exists.

If $B_0 \in (0, B_-)$, strict monotonicity and (D.4) give $B_t < B_{t+1} < B_-$ at every date along the live path. The sequence increases to a positive fixed point, necessarily B_- . If $B_0 \in (B_-, B_+)$, the same argument gives $B_- < B_{t+1} < B_t$, so the sequence decreases to B_- . The cases $B_0 = B_-$ and $B_0 = B_+$ are stationary.

Finally, let $B_0 \in (B_+, e_1)$. If a feasible continuation exists, it is strictly greater than the current price. Suppose, toward a contradiction, that feasible continuations exist at every date. Then the price sequence is increasing and bounded above by e_1 , and therefore converges to some $\ell \in (B_+, e_1]$.

If $\ell < e_1$, continuity of the sponsor best response implies $q_t = \mathbf{q}(B_{t+1}) \rightarrow \mathbf{q}(\ell) > 0$. Taking limits in household pricing gives

$$u'(e_1 - \ell) = \gamma \mathbf{q}(\ell) v'(e_2 + \ell).$$

Thus $(\ell, \mathbf{q}(\ell))$ is a stationary equilibrium above B_+ , a contradiction. This argument uses the equilibrium conditions directly and does not require T to be defined at a boundary of its domain.

If $\ell = e_1$, household pricing instead implies

$$B_t u'(e_1 - B_t) = \gamma q_t B_{t+1} v'(e_2 + B_{t+1}) \leq \gamma e_1 v'(e_2) < \infty,$$

contradicting the Inada condition as $B_t \uparrow e_1$. Therefore an initial price above B_+ either admits no feasible continuation immediately or ceases to admit one after finitely many iterations. Such a price cannot be part of an infinite-horizon positive live equilibrium. $\square$

D.4 The governance externality

This proof returns to the baseline linear-old-age-utility comparison.

Proof of Proposition 2. The two optimization problems differ only in the coefficient on the continuation value of survival.

Because

$$s\gamma B < \gamma B$$

when $s < 1$, and because the objective has increasing differences in $(q, \gamma B)$, the optimal maintenance choice is weakly increasing in the shadow value of survival. Therefore,

$$\mathbf{q}(B) \leq q^A(B).$$

If $\gamma B > \psi'(0)$, the all-owner solution is positive. Strict convexity of ψ and the strictly smaller sponsor coefficient then imply $\mathbf{q}(B) < q^A(B)$, whether the sponsor is at the zero corner or in the interior.

For the value comparison, the Fenchel conjugate representation gives

$$V^I(B) = \psi^*(s\gamma B), \quad V^A(B) = \psi^*(\gamma B).$$

Since the convex conjugate is weakly increasing in its argument,

$$V^A(B) \geq V^I(B).$$

It is strictly increasing once its optimizer is positive, which gives the stated strict value

comparison under $\gamma B > \psi'(0)$. □

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